

IIM CAT · QUANT

Quantitative Aptitude Notes & Short Tricks

Handwritten Quant notes plus a shortcut compendium to save time on test day.

Curated for CAT aspirants

Free download

Q1. In January of the year 1900, I was one more than eleven times as old as my son William. In January of 1909, I was seven more than three times as old as him. How old was my son in January 1900?

(a) 3

(b) 6

(c) 4

(d) 7

Ans = (a), 3.

 $a = \text{me}$ $b = \text{William}$

year 1900,

$$a = 11b + 1 \quad - (1)$$

year 1909,

$$a + 9 = 3(b + 9) + 7 \quad - (2)$$

in eqⁿ (1) & (2) $a = 11b + 1$

$$(11b + 1) + 9 = 3(b + 9) + 7$$

$$11b - 3b = 34 - 9 - 1$$

$$8b = 24$$

$$b = 3$$

 $\therefore b = \text{William} = \underline{3}$

Q 2. In a school of 600 boys and 400 girls and the average scholarship was Rs. 25.50 per day. If a girl got Rs. 5 less than a boy, then what are their daily scholarship amount?

(a) boy = Rs. 25 ; girl = Rs. 20

(b) boy = Rs. 27.50 ; girl = Rs. 22.50

(c) boy = Rs. 30 ; girl = Rs. 25

(d) boy = Rs. 32.50 ; girl = Rs. 27.50

Ans = (b), boy = Rs. 27.50 ; girl = Rs. 22.50

Let the daily scholarship of a boy be Rs. x .
Then, daily scholarship of a woman = Rs. $(x - 5)$

$$600x + 400(x - 5) = 25.50 \times (600 + 400)$$

$$1000x = 27500$$

$$x = 27.50$$

$$\text{boy} = x = 27.50$$

$$\text{girl} = (x - 5) = 27.50 - 5 = 22.50$$

Q3 The average age of employees of a company is 15.8 years. The average age of males in the company is 16.4 years and that of females is 15.4 years. The ratio of the number of males to the number of females in the company is ?

(a) 1:2

(b) 3:4

(c) 3:5

(d) 2:3

Ans = (d), 2:3

$$\text{males} \therefore \text{females} = k:1$$

$$K \times 16.4 + 1 \times 15.4 = (K+1) \times 15.8$$

$$(16.4 - 15.8)K = 15.8 - 15.4$$

$$K = \frac{0.4}{0.6} = \frac{2}{3}$$

$$\text{Required ratio} = \frac{2}{3} : 1$$

$$\therefore \underline{2:3}$$

Q4. A company plans a salary cut for all its employees. The current salary of the employee is 1600. ~~65%~~ 65% employees take away Rs. 40,000 per month each, 22.5% employees takes away Rs. 25,000 per month each and 12.5% employees take away Rs. 15,000 per month each. If all the employees have to suffer a 10% cut from the salary for a month. What is the salary cut of the whole company for $\pm$ 1 month?

(a) Rs. 53.6×10^4

(b) Rs. 5.36×10^3

(c) Rs. 536×10^4

(d) None.

Ans = (c), Rs. 536×10^4 .

Let, $12.5\% = \frac{1}{8} = \frac{5}{40}$

$22.5\% = \frac{9}{40}$

$65\% = \frac{13}{20} = \frac{26}{40}$

$\therefore$ The total salary 'S' be,

$$S = \left(\frac{5}{40}\right) \times 1600 \times 15000 + \left(\frac{9}{40}\right) \times 1600 \times 25000 + \left(\frac{26}{40}\right) \times 1600 \times 40000$$

5

$$= 40 \times 1000 \times (75 + 225 + 1040)$$

$$S = 5360 \times 10^4$$

$\therefore$ Salary cut off is 10%.

$$5360 \times 10^4 \times \frac{10}{100}$$

$$= 536 \times 10^4$$

— •

Q.5

In an election, 30% of the voters voted for candidate A whereas 60% of the remaining voted for candidate B.

The remaining voters did not vote. If the difference between those who voted for candidate A and those who did not vote was 1200, how many individuals were eligible for casting vote in that election?

(a) 16,000

(b) 40,000

(c) 60,000

(d) 54,000

Ans = (c), 60,000

Let the number of persons eligible to vote be x .

voters who voted for A = 30% of x

voters who voted for B = 60% of (70% of x)

$$= \left(\frac{60}{100} \times \frac{70}{100} \times 100 \right) \% \text{ of } x$$

$$= 42 \% \text{ of } x$$

$$\begin{aligned} \text{voters who did not vote} &= [100 - (30 + 42)] \% \text{ of } x \\ &= 28 \% \text{ of } x \end{aligned}$$

7

$$\therefore 30\% \text{ of } x - 28\% \text{ of } x = 1200$$

$$2\% \text{ of } x = 1200$$

$$x = \frac{1200 \times 100}{2}$$

$$x = 60,000$$

Q6. Rahul and Hardik invested the same amount of money in different schemes. But as per the scheme where Rahul invested, the money grows to $196/169$ times when invested for 2 years where interest was compounded annually. whereas Hardik was suggested a scheme in which his money would become 5 times if invested at the same rate of interest as that of Rahul with interest being computed using simple interest method. Then approximately how many years would be needed to do so?

(a) 25

(b) 52

(c) 40

(d) 39

Ans = (b), 52.

In compound interest,

if a sum is invested for 2 years, then the amount after 2 years is given,

$$A = P \times \left(1 + \frac{R}{100}\right)^2$$

$$A = \left(\frac{196}{169}\right) P$$

9

$$\left(1 + \frac{R}{100}\right)^2 = \frac{196}{169}$$

$$R = \left(\frac{14}{13} - 1\right) \times 100$$

$$R = \frac{100}{13}$$

In simple interest, if a sum is invested for 'N' years, then the amount after 'N' years is given by,

$$A = P + \frac{P \times N \times R}{100}$$

$$SP = P + P \times N \times \frac{1}{13}$$

$$4 = \frac{N}{13}$$

$$N = \underline{52} \text{ years}$$

Q7. Anil borrowed some money from John at 12% p.a Simple interest for 3 years. He then added some more money to the borrowed sum and lent it to David for the same period at 14% p.a rate of interest. If Anil gains Rs. 93.90 in the whole transaction, how much money did he add from his side?

(a) 105

(b) 100

(c) 90

(d) 95

Ans = (a), 105.

Let the money added be x .

Then,

$$\frac{(830 + x) \times 14 \times 3}{100} - \frac{830 \times 12 \times 3}{100} = 93.90$$

$$830 \times 42 + 42x - 830 \times 36 = 9390$$

$$42x + 830 \times (42 - 36) = 9390$$

$$42x = 9390 - 4980$$

$$x = \frac{4410}{42} = 105$$

11
98. The difference between compound interest and simple interest on a sum for 2 years at 10% per annum, when the interest is compounded annually is Rs. 16. If the interest were compounded half yearly, the difference in two interests will be,

(a) Rs. 40.51

(b) Rs. 20.00

(c) Rs. 24.81

(d) Rs. 36.41

Ans = (c), Rs. 24.81.

For first year S.I. = C.I.

Now, Rs. 16 is the S.I. on S.I. for 1 yr.
Rs. 10 is S.I. on Rs. 100.

$$\therefore \text{Rs. 16 is S.I. on Rs. 1000 } \text{Rs.} \left(\frac{100}{10} \times 16 \right) = \text{Rs. 160.}$$

So, S.I. on principal for 1 year at 10% is Rs. 160.

$$\therefore \text{Principal} = \text{Rs.} \left(\frac{100 \times 160}{10 \times 1} \right) = \text{Rs. 1600.}$$

$$\text{Amount for 2 years compounded half yearly} = \text{Rs.} \left[1600 \times \left(1 + \frac{5}{100} \right)^4 \right].$$

$$= \text{Rs. 1944.81}$$

12

$$C.I = Rs (1944.81 - 1600)$$

$$= Rs 344.81$$

$$S.I = Rs \left(\frac{1600 \times 10 \times 2}{100} \right) =$$

$$= Rs 320.$$

$$C.I - S.I = 344.81 - 320$$
$$= \underline{24.81}$$

99. Rohil ~~tells~~ has is selling wheat at price of Rs. 12 per kg after discount at Rs. 10 per kg. He was mixing lower quality wheat of price Rs. 6 per kg in some portion and thus makes a profit of 20%. What is the proportion in which the good quality and low quality wheat are mixed and sold to customers?

(a) $\frac{9}{4}$

(b) $\frac{7}{11}$

(c) $\frac{17}{12}$

(d) $\frac{14}{5}$

Ans = (b) $\frac{7}{11}$

At a selling price of Rs. 10 per kg, profit is 20%.

Actual cost price = $\frac{10}{1.2}$

Let there be x kg of Rs. 12 ^{wheat} and y kg of Rs. 6 ^{wheat}.

Cost price = $\frac{12x + 6y}{x + y} = \frac{10}{1.2}$

$\therefore \frac{x}{y} = \frac{7}{11}$

Q.10. 85 kg of a mixture contains petrol and water in the ratio 27:7. How much more water is to be added to get a new mixture containing petrol and water in the ratio 3:1?

(a) 10 kg

(b) 6 kg

(c) 5 kg

(d) 14 kg

Ans = (c), 5 kg.

Petrol in 85 kg of mixture,

$$\left(85 \times \frac{27}{34}\right) \text{ kg} = \frac{135}{2} \text{ kg}.$$

$$\text{Water in it} = \left(85 - \frac{135}{2}\right) \text{ kg} = \frac{35}{2} \text{ kg}.$$

Let = x

Let x kg of water be added to it.

$$\frac{\left(\frac{135}{2}\right)}{\left(\frac{35}{2} + x\right)} = \frac{3}{1}$$

$$\frac{135}{35 + 2x} = \frac{3}{1}$$

15

$$105 + 6x = 135$$

$$6x = 30$$

$$x = 5 \text{ kg}$$

—————

Q.11. When three brands of orange juice are mixed in the ratio of 3:4:5 and 4:5:6, the cost price of the mixture of orange juice comes out to be Rs. 20 and Rs. 25 per litre respectively. What will be the cost price of a litre of mixture in which the three brands are mixed in the ratio 6:7:8?

(a) 35.35

(b) 21.46

(c) 30.70

(d) 11.29

Ans = (c), 30.70

Let a, b, c be the cost price of three brands of orange juice per litre,

$$\therefore \frac{3a + 4b + 5c}{12} = 20$$

$$3a + 4b + 5c = 240$$

$$\frac{4a + 5b + 6c}{15} = 25$$

$$4a + 5b + 6c = 375$$

17

$$6a + 7b + 8c = 3 \times [4a + 5b + 6c] - 2 \times [3a + 4b + 5c]$$

$$= 3 \times 375 - 2 \times 240$$

$$= 645.$$

Cost price of mixture of orange juice mixed in the ratio 6:7:8 per litre.

$$6 + 7 + 8 = 21$$

$$= \frac{645}{21} = \underline{\underline{Rs. 30.70}}$$

Q.12 Riya travels from Mumbai to Pune a distance of 250 miles in $5\frac{1}{2}$ hrs. She returns to Mumbai at a speed of ~~2 km/hr~~ in 4 hrs 30 mins. Her average speed is.

(a) 45 mph

(b) 33 mph

(c) 60 mph

(d) 50 mph

Ans = (d), 50 mph.

Speed from Mumbai to Pune,

$$\left(250 \times \frac{2}{11}\right) \text{ mph} = \left(\frac{500}{11}\right) \text{ mph}.$$

Speed from Pune to Mumbai,

$$\left(250 \times \frac{2}{9}\right) \text{ mph} = \left(\frac{500}{9}\right) \text{ mph}.$$

$$\therefore \text{Average speed} = \left(\frac{2 \times \frac{500}{11} \times \frac{500}{9}}{\frac{500}{11} + \frac{500}{9}} \right) \text{ mph}$$

$$= \left(\frac{500000}{4500 + 5500} \right) \text{ mph} = 50 \text{ mph}.$$

Q.13.

In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. The duration of the flight is —.

(a) 3 hrs

(b) 1 hr

(c) 4 hrs

(d) 2 hrs

Ans = (b), 1 hr.

Let the duration of the flight be x hrs.

$$\frac{600}{x} = \frac{600}{x + \frac{1}{2}} = 200$$

$$\frac{600}{x} - \frac{1200}{2x+1} = 200$$

$$x(2x+1) = 3$$

$$2x^2 + x - 3 = 0$$

$$(2x+3)(x-1) = 0$$

$$x = \underline{1 \text{ hr}}$$

Q14.

A train 110 metres long is running with a speed of 60 kmph. In what time will it pass a man who is running at 6 kmph in the direction opposite to that in which the train is going?

(a) ~~8~~ 8 sec

(b) 6 sec

(c) 10 sec

(d) 5 sec

Ans = (b), 6 sec.

Speed of train relative to man,

$$(60 + 6) \text{ km/hr} = 66 \text{ km/hr}$$

$$= \left(66 \times \frac{5}{18} \right) \text{ m/sec}$$

$$= \left(\frac{55}{3} \right) \text{ m/sec.}$$

$$\therefore \text{Time taken to pass the man} = \left(110 \times \frac{3}{55} \right) \text{ sec}$$

$$= \underline{6 \text{ sec.}}$$

Q15. A jogger running at 9 kmph alongside a railway track is 240 metres ahead of the engine of a 120 metre long train running at 45 kmph in the same direction. In how many much time will the train pass the jogger?

(a) 40 sec

(b) 36 sec

(c) 15 sec

(d) 20 sec.

Ans = (b), 36 sec.

Speed of train relative to jogger = $45 - 9 = 36 \text{ km/hr}$

$$= \left(\frac{36 \times 5}{18} \right) \text{ m/sec} = 10 \text{ m/sec}.$$

Distance to be covered = $(240 + 120) \text{ m}$
 $= 360 \text{ m}.$

$$\therefore \text{Time taken} = \left(\frac{360}{10} \right) \text{ sec} = 36$$

36 sec

Q16. Three pipes A, B and C can fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is.

(a) 14

(b) 10

(c) 20

(d) 16

Ans = (a), 14.

$$\text{Part filled in 2 hours} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Remaining part} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\text{Work done by A \& B in 7 hours} = \frac{2}{3}$$

$$\text{work done by A \& B in 1 hour} = \frac{2}{21}$$

$$\begin{aligned} \text{work done by C in 1 hour} &= \\ &= \left[(A+B+C) \text{ 1 hr work} - (A+B) \text{ 1 hr work} \right] \\ &= \frac{1}{6} - \frac{2}{21} = \frac{1}{14} \end{aligned}$$

C can fill the tank in 14 hrs

Q17. Two pipes A and B can independently fill a cistern in 20 minutes and 25 minutes respectively. Both are opened together for 5 minutes, after which B is turned off. How much more time will the cistern take to fill?

(a) 23

(b) 12

(c) 11

(d) 19

Ans = (c), 11.

A and B can fill the cistern alone in 20 minutes and 25 minutes respectively.

∴ In one minute, they can together fill $\left(\frac{1}{20}\right) + \left(\frac{1}{25}\right) = \left(\frac{9}{100}\right)$ of the cistern.

∴ In 5 minutes, they can fill $\frac{9}{20}$ of the cistern.

After 5 minutes, B is turned off, so the remaining $\frac{11}{20}$ of the cistern is to be filled alone by A.

A takes 20 minutes to fill up the entire cistern, It will take 11 more minutes.

Q.18. A boat covers a certain distance downstream in 1 hour, while it comes back in $1\frac{1}{2}$ hrs. If the speed of the stream be 3 kmph, what is the speed of the boat in still water?

(a) 11

(b) 15

(c) 10

(d) 18

Ans = (b) , 15.

Let the speed of the boat in still water be x kmph.

Speed downstream = $(x+3)$ kmph.

Speed upstream = $(x-3)$ kmph.

$$(x+3) \times 1 = (x-3) \times \frac{3}{2}$$

$$2x+6 = 3x-9$$

$$x = 15 \text{ kmph.}$$

819 At his usual rowing rate, Rahul can travel 12 miles downstream in a certain river in 6 hrs less than it takes him to travel the same distance upstream. But if he could double his usual rowing rate for his 24-mile round trip, the downstream 12 miles would then take only one hour less than the upstream 12 miles. What is the speed of the current in miles per hour?

(a) $1\frac{2}{3}$

(b) $4\frac{1}{3}$

(c) $2\frac{2}{3}$

(d) $5\frac{3}{2}$

Ans = (c) $2\frac{2}{3}$.

Let the speed in still water be x mph and the speed of the current be y mph.

Speed upstream = $(x - y)$

Speed downstream = $(x + y)$

$$\frac{12}{(x - y)} - \frac{12}{(x + y)} = 6$$

$$6(x^2 - y^2) = 24y$$

$$x^2 - y^2 = 4y$$

$$x^2 = (4y + y^2) \quad \text{--- (1)}$$

$$\frac{12}{(2x-y)} - \frac{12}{(2x+y)} = 4$$

$$4x^2 - y^2 = 24y$$

$$x^2 = \frac{24y + y^2}{4} \quad \text{--- (2)}$$

from eqⁿ (1) & (2)

$$4y + y^2 = \frac{24y + y^2}{4}$$

$$16y + 4y^2 = 24y + y^2$$

$$3y^2 = 8y$$

$$y = \frac{8}{3}$$

$\therefore$ Speed of the current = $\frac{8}{3}$ mph

$$= 2\frac{2}{3} \text{ mph}$$

Q.20 Cost of 3 Apples, 6 Oranges and 5 Bananas is same as cost of 5 Apples, 2 Oranges and 7 bananas, which in turn is equal to one third of the cost of 17 Apples, 2 Oranges and 23 Bananas. If 7 Apples, 2 Oranges and 23 Bananas together cost Rs. 132. Then what is the cost of 12 Apples and 16 Bananas?

(a) 88

(b) 90

(c) 120

(d) 118.

Ans = (a), 88.

Let the cost of 1 Apple, 1 Orange & 1 Banana be a , b and c respectively.

$$17a + 2b + 23c = 132$$

$$3a + 6b + 5c = 44 \quad \text{--- (1)}$$

$$5a + 2b + 7c = 44 \quad \text{--- (2)}$$

$$\text{Now } 12a + 6c = ?$$

multiply eqⁿ (2) by 3 and then subtract
eqⁿ (1) from it,

$$(15-3)a + (6-6)b + (21-5)c = 44(3-1)$$

$$12a + 16c = \underline{88}.$$

Q.21. Jaya, Megha, Abhay, Rohit and Harsha contested an election. There are 1200 valid voters. Rohit got half the number of votes that Jaya and Megha got. Abhay got half of the votes that the other four got. Jaya, Abhay, Rohit and Harsha's votes together were thrice the number of votes that Megha got. Jaya got one-fifth the votes that the others got. How many votes more than Harsha did Rohit get?

(a) 150

(b) 160

(c) 200

(d) 100

Ans = (c), 200.

Let Jaya, Megha and Abhay get a, b, c votes respectively.

$\therefore$ Jaya got one-fifth the votes that the others got,

$$5a = 1200 - a$$

$$a = 200.$$

$\therefore$ Jaya, Abhay, Rohit and Harsha votes together were thrice the no. of votes that Megha got,

30

$$3b = 1200 - b$$

$$b = 300$$

Abhay got half of the votes that the other four got,

$$2c = 1200 - c$$

$$c = 400$$

Rohit got half the number of votes that Jaya and Megha got,

$$\text{No. of votes Rohit got} = \frac{200 + 300}{2} = 250 \text{ votes}$$

$$\begin{aligned} \text{Harsha got} &= 1200 - 200 - 300 - 400 - 250 \\ &= 50 \text{ votes} \end{aligned}$$

Rohit got $(250 - 50) = 200$ more votes than Harsha.

3

Q22. Wheels of diameter 7 cm and 14 cm start rolling simultaneously from A and B, which are 1980 cm apart, towards each other in opposite directions. Both of them make the same number of revolutions per second. If both of them meet after 10 seconds, the speed of the smaller is —.

(a) 64 m/sec

(b) 22 m/sec

(c) 40 m/sec

(d) 18 m/sec

Ans = (b), 22 m/sec.

Let each wheel make x revolution per sec.

$$\left[\left(2\pi \times \frac{7}{2} \times x \right) + \left(2\pi \times 7 \times x \right) \right] \times 10 = 1980$$

$$\left(\frac{22}{7} \times 7 \times x \right) + \left(2 \times \frac{22}{7} \times 7 \times x \right) = 198$$

$$66x = 198$$

$$x = 3$$

Distance moved by smaller wheel in 3 revolution = $2 \times \frac{22}{7} \times \frac{7}{2} \times 3 = 66 \text{ cm}$.

$\therefore$ speed of smaller wheel = $\frac{66}{3} \text{ m/sec} = 22 \text{ m/sec}$.

Q.23 If a wire is bent into the shape of a square, then the area of the square is 81 sq. cm . When the wire is bent into a semi-circular shape, then the area of the semi-circle will be _____

(a) 77 cm^2

(b) 80 cm^2

(c) 149 cm^2

(d) 50 cm^2

Ans = (a), 77 cm^2

Length of each side of the square = $\sqrt{81}$
= 9 cm

Length of wire = $(9 \times 4) \text{ cm} = 36 \text{ cm}$

$\pi R + 2R = 36$

$(\pi + 2)R = 36$

$R = \frac{36}{\left(\frac{22}{7} + 2\right)} = 7 \text{ cm}$

$\left(\frac{22}{7} + 2\right)$

Area of the semi-circle = $\frac{1}{2} \pi R^2$

= $\left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right) \text{ cm}^2$

= 77 cm^2

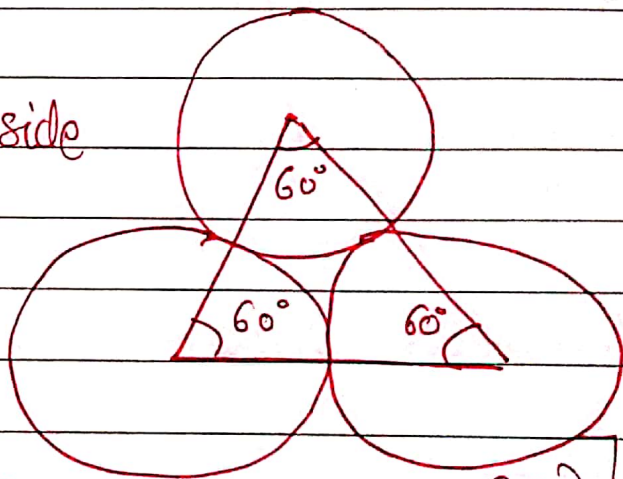
Q.24. Three circles of radius 3.5 cm are placed in such a way that each circle touches the other two. The area of the portion enclosed by the circles is —

- (a) 19.67 cm^2 (b) 2.410 cm^2
 (c) 3.141 cm^2 (d) 1.967 cm^2

Ans = (d), 1.967 cm^2

Required area =

(Area of equilateral Δ of side 7 cm) - (3 $\times$ area of sector with $\theta = 60^\circ$).



$$= \left[\frac{\sqrt{3}}{4} \times 7 \times 7 \right] - \left(3 \times \frac{22}{7} \times 3.5 \times 3.5 \times \frac{60}{360} \right) \text{ cm}^2$$

$$= \left(\frac{49\sqrt{3}}{4} - 11 \times 0.5 \times 3.5 \right) \text{ cm}^2$$

$$= (21.217 - 19.25) \text{ cm}^2$$

$$= \underline{1.967 \text{ cm}^2}$$

Q25. The length and breadth of the floor of the room are 20 feet and 10 feet respectively.

Square tiles of 2 feet length of different colours are to be laid on the floor.

Black tiles are laid in the first row on all sides. If white tiles are laid in the one-third of the remaining and blue tiles in the rest, how many blue tiles will be there?

(a) 18

(b) 16

(c) 24

(d) 50

Ans = (b), 16.

Area left after laying black tiles

$$= \sqrt{[(20-4) \times (10-4)]} \text{ sq. ft} \\ = 96 \text{ sq. ft.}$$

$$\text{Area under white tiles} = \left(\frac{1}{3} \times 96\right) \text{ sq. ft} = 32 \text{ sq. ft}$$

$$\text{Area under blue tiles} = 96 - 32 = 64 \text{ sq. ft.}$$

$$\text{No. of blue tiles} = \frac{64}{4} = \underline{16}$$

Q.26 The percentage increase in the area of a rectangle, if each of its sides is increased by 20% is —.

(a) 44 %

(b) 49 %

(c) 35 %

(d) 40 %

Ans = ~~(a)~~ (a), 44 %

Let original length = x meters

original length = y meters

original area = $(xy) \text{ m}^2$

$$\text{New length} = \left(\frac{120}{100} x \right) \text{ m} = \left(\frac{6}{5} x \right) \text{ m}$$

$$\text{New breadth} = \left(\frac{120}{100} y \right) \text{ m} = \left(\frac{6}{5} y \right) \text{ m}$$

$$\text{New Area} = \left(\frac{6}{5} x \times \frac{6}{5} y \right) \text{ m}^2 = \left(\frac{36}{25} xy \right) \text{ m}^2$$

∴ Increased Percentage

$$= \left(\frac{11}{25} xy \times \frac{1}{xy} \times 100 \right) \%$$

$$= 44\%$$

Q27 John, Mary and Darry enter into a partnership in the ratio $7\frac{1}{2} : 4\frac{1}{3} : 6\frac{1}{5}$.

After 4 months, John increases his share by 50%. If the total profit at the end of one year be Rs. 21,600, then Mary's share in the profit is ?

(a) 4200

(b) 2000

(c) 6000

(d) 4000

Ans = (d), 4000.

Ratio of initial investments = $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$

$= 105 : 40 : 36$

Let the initial investments be

$105x$, $40x$ and $36x$.

John : Mary : Darry = $105x : 40x : 36x$

$= \left(105x \times 4 + \frac{150}{100} \times 105x \times 8 \right) :$

$(40x \times 12) : (36x \times 12)$

$= 1680x : 480x : 432x$

$= 35 : 10 : 9 = 35 + 10 + 9 = 54$

Mary's share = $\frac{21600 \times 10}{54} = \underline{4000}$

37

Q28 Reena and Asha entered into partnership with capitals in the ratio 4:5. After 3 months, Reena withdrew $\frac{1}{4}$ of her capital and Asha withdrew $\frac{1}{5}$ of her capital. The gain at the end of 10 months was Rs. 760. What is Reena's share in the profit?

(a) 400

(b) 300

(c) 330

(d) 350

∴ Ans = (c), 330.

Reena : Asha

$$= \left[4x \times 3 + \left(4x - \frac{1}{4} \times 4x \right) \times 7 \right] : \left[5x \times 3 + \left(5x - \frac{1}{5} \times 5x \right) \times 7 \right]$$

$$= (12x + 21x) : (15x + 28x)$$

$$= 33x : 43x$$

$$= 33 : 43$$

$$33 + 43 = 76$$

$$\therefore \text{Reena's share} = 760 \times \frac{33}{76}$$

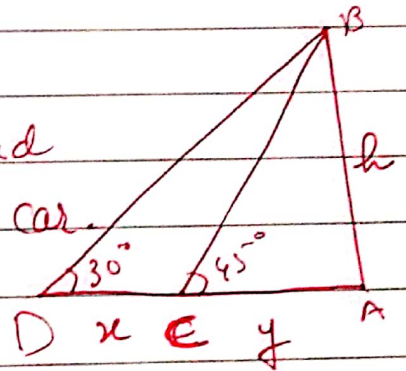
$$= 330$$

8.29. A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45° , how soon after this will the car reach the observation tower?

- (a) 15 min 42 sec (b) 16 min 32 sec
 (c) 16 min 23 sec (d) 11 min 40 sec

Ans = (c), 16 min 23 sec.

Let AB be the tower and C and D be the two positions of the car.



$$\angle ACB = 45^\circ, \angle ADB = 30^\circ$$

Let

$$AB = h, CD = x, AC = y.$$

$$\frac{AB}{AC} = \tan 45^\circ = 1$$

$$\frac{h}{y} = 1 \therefore y = h$$

$$\frac{AB}{AO} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\frac{h}{x+y} = \frac{1}{\sqrt{3}}$$

$$\therefore x+y = \sqrt{3}h$$

~~1600~~, ~~11(1.3+1)~~

39

$$\therefore x = (x + y) - y = \sqrt{3}h - h \\ = h(\sqrt{3} - 1)$$

Now, $h(\sqrt{3} - 1)$ is covered in 12 min.

$$\text{So, } h \text{ will be covered ? in } \left(\frac{12}{h(\sqrt{3} - 1)} \times h \right) \\ = \frac{12}{(\sqrt{3} - 1)} \text{ min}$$

$$= \left(\frac{1200}{73} \right) \text{ min}$$

$$= \underline{16 \text{ min}} . 23 \text{ sec.}$$

Q30. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30° . What is the approximate speed of the boat, assuming that it is running in still water?

(a) 35 km/hr

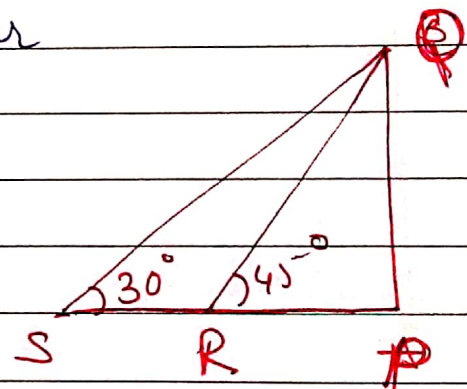
(b) 24 km/hr

(c) 42 km/hr

(d) 32 km/hr

Ans = (d), 32 km/hr

Let PQ be the tower, S and R be the two positions of the boat.



$$\angle PRQ = 45^\circ, \angle PSQ = 30^\circ, PR = 60 \text{ m}$$

$$\text{Let } PQ = h$$

$$\text{The } \frac{PQ}{PR} = \tan 45^\circ = 1$$

$$PQ = PR = h = 60 \text{ m}$$

44

$$\frac{PQ}{PS} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$PS = PQ \times \sqrt{3} = 60\sqrt{3} \text{ m}$$

$$\therefore SR = (PS - PR) = 60(\sqrt{3} - 1) \text{ m}$$

$$\text{Required speed} = \left[\frac{60(\sqrt{3} - 1)}{5} \right] \text{ m/sec}$$

$$= (12 \times 0.73) \text{ m/sec}$$

$$= \left(12 \times 0.73 \times \frac{18}{5} \right) \text{ km/hr}$$

$$= 31.5 \text{ km/hr}$$

$$= \underline{32 \text{ km/hr}}$$

QUALITATIVE APTITUDE TRICKS & SHORTCUTS FOR CAT

Celebrating Excellence Across Spectrum

Jaipuria Institute of Management
Ranked Amongst the Top
Management Institutes



NAAC^{**}
Accredited

Graded Autonomy
by **AICTE**^{^^}

NBA[^]
Accredited

AIU[^] Recognised PGDM
equivalent to MBA

AACSB
Business
Education
Alliance
Member

Index

- | | |
|--|--|
| 1. Logarithms, surds and indices | 70. Geometry/Mensuration for CAT |
| 9. Permutations and combinations | 111. Time, Speed, Distance & Work |
| 19. Simple interest and compound interest | 128. Number Systems |
| 27. Profit & Loss, Discount | 148. Ratio & Proportion |
| 35. Set theory and Venn diagrams | 159. Inequalities |
| 47. Mixture & Alligations | 165. Quadratic Equations |
| 53. Progressions and Series | 174. Linear Equations |

Tip 1 – Logarithms, Surds and Indices

- “Logarithms, Surds and Indices” is one of the easiest topics in the quantitative section of the CAT exam.
- Although the number of formulae is high, the basic concepts are very simple to understand and apply.
- There are no shortcuts to remember and the scope of the questions that can be asked is very limited.
- The accuracy of answering questions from this section is very high and good students tend to score very well here.

Tip 2 – Logarithms, Surds and Indices

If $X, Y > 0$ and m, n are rational numbers then

- $X^m \times X^n = X^{m+n}$
- $X^0 = 1$
- $\frac{X^m}{X^n} = X^{m-n}$
- $(X^m)^n = X^{mn}$
- $X^m \times Y^m = (X \times Y)^m$
- $\frac{X^m}{Y^m} = (X/Y)^m$
- $X^{-m} = \frac{1}{X^m}$

Tip 3 – Logarithms, Surds and Indices

If X and Y are positive real numbers and a,b are rational numbers.

- $(X/Y)^{-a} = (Y/X)^a$
- $X^{1/a} = \sqrt[a]{X}$
- $X^{a/b} = \sqrt[b]{X^a}$
- $\sqrt[a]{X} \times \sqrt[a]{Y} = \sqrt[a]{XY}$
- $\sqrt[a]{X} / \sqrt[a]{Y} = \sqrt[a]{X/Y}$
- $\frac{1}{\sqrt{N+1}-\sqrt{N}} = \sqrt{N+1} + \sqrt{N}$

Tip 4 – Logarithms, Surds and Indices

- Surd is an irrational number involving a root ex : $\sqrt{5}$, $\sqrt[3]{7}$, $\sqrt[5]{2}$
- Like surds are two surds having same number under radical sign.
- Like surds can be added or subtracted. $6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$

Tip 5 – Logarithms, Surds and Indices

- If $a+\sqrt{b} = c+\sqrt{d}$, then $a = c$ and $b = d$
- The conjugate of $a+\sqrt{b}$ is $a-\sqrt{b}$
- $\sqrt{a\sqrt{a\sqrt{a\ldots\infty}}} = a$
- $\sqrt{a\sqrt{a\sqrt{a\ldots x \text{ times}}}} = a^{1-[1/(2^x)]}$
- To find $\sqrt{\sqrt{x} + \sqrt{y}}$, $\sqrt{x} + \sqrt{y}$ should be written in the form of $m+n+2\sqrt{mn}$ where $x = m+n$ and $4mn = y$ and $\sqrt{\sqrt{x} + \sqrt{y}} = \pm (\sqrt{m} + \sqrt{n})$

Tip 6 – Logarithms, Surds and Indices

If $N = a^x$ then, x is defined as the logarithm of N to base a
or $x = \log_a N$

Logarithm of a negative number or zero is not defined.

- $\log_a 1 = 0$
- $\log_a xy = \log_a x + \log_a y$
- $\log_a b^c = c \log_a b$
- $\log_a a = 1$
- $x^{\log_b y} = y^{\log_b x}$

Tip 7 – Logarithms, Surds and Indices

- $\log_a \sqrt[n]{b} = \frac{\log_a b}{n}$
- $\log_a x = \frac{1}{\log_x a}$
- $b^{\log_b x} = x$
- $\log_a b = \frac{\log_c b}{\log_c a}$
- $\log_a b * \log a = 1$
- $\log_a (X/Y) = \log_a X - \log_a Y$

Tip 8 – Logarithms, Surds and Indices

- If $0 < a < 1$, then $\log_a x < \log_a y$ (if $x > y$)
- If $a > 1$ then $\log_a x > \log_a y$ (if $x > y$)

Tip 1 – Permutations & Combinations

- Permutations & Combinations is an extremely important topic in CAT.
- This topic can be the most rewarding topic in quant section.
- Unlike number systems questions, most of these questions generally take lesser time to solve.
- Also, they are generally fairly basic in nature.

Tip 2 – Permutations & Combinations

- The more questions you solve, the better you will get at this topic.
- So look through the formula list a few times and understand the formulae.
- But the best way to tackle this subject is by solving questions.
- Solve as many questions as you can, from this topic that you will start to see that all of them are generally variations of the same few themes that are listed in the formula list.

Tip 3 – Permutations & Combinations

- $N! = N(N-1)(N-2)(N-3)\dots\dots 1$
- $0! = 1! = 1$
- ${}^nC_r = \frac{n!}{(n-r)! r!}$
- ${}^nP_r = \frac{n!}{(n-r)!}$

Tip 4 – Permutations & Combinations

- **Arrangement :**

n items can be arranged in $n!$ ways

- **Permutation :**

A way of selecting and arranging r objects out of a set of n objects, ${}^n P_r = \frac{n!}{(n-r)!}$

- **Combination :**

A way of selecting r objects out of n (arrangement does not matter)

$${}^n C_r = \frac{n!}{(n-r)!r!}$$

Tip 5 – Permutations & Combinations

- Selecting r objects out of n is same as selecting $(n-r)$ objects out of n , ${}^nC_r = {}^nC_{n-r}$
- Total selections that can be made from ' n ' distinct items is given $\sum_{k=0}^n {}^nC_k = 2^n$

Tip 6 – Permutations & Combinations

Partitioning :

- Number of ways to partition n identical things in r distinct slots is given by ${}^{n+r-1}C_{r-1}$
- Number of ways to partition n identical things in r distinct slots so that each slot gets at least 1 is given by ${}^{n-1}C_{r-1}$
- Number of ways to partition n distinct things in r distinct slots is given by r^n
- Number of ways to partition n distinct things in r distinct slots where arrangement matters = $\frac{(n+r-1)!}{(r-1)!}$

Tip 7 – Permutations & Combinations

Arrangement with repetitions :

If x items out of n items are repeated, then the number of ways of arranging these n items is $\frac{n!}{x!}$ ways. If a items, b items and c items are repeated within n items, they can be arranged in $\frac{n!}{a! b! c!}$ ways.

Tip 8 – Permutations & Combinations

Rank of a word :

- To get the rank of a word in the alphabetical list of all permutations of the word, start with alphabetically arranging the n letters. If there are x letters higher than the first letter of the word, then there are at least $x \cdot (n-1)!$ Words above our word.
- After removing the first affixed letter from the set if there are y letters above the second letter then there are $y \cdot (n-2)!$ words more before your word and so on. So rank of word = $x \cdot (n-1)! + y \cdot (n-2)! + \dots + 1$

Tip 9 – Permutations & Combinations

Integral Solutions :

- Number of positive integral solutions to $x_1+x_2+x_3+.....+x_n = s$ where $s \geq 0$ is ${}^{s-1}C_{n-1}$
- Number of non-negative integral solutions to $x_1+x_2+x_3+.....+x_n = s$ where $s \geq 0$ is ${}^{n+s-1}C_{n-1}$

Tip 10 – Permutations & Combinations

Circular arrangement :

Number of ways of arranging n items around a circle are 1 for $n = 1, 2$ and $(n-1)!$ for $n \geq 3$. If its a necklace or bracelet that can be flipped over, the possibilities are $\frac{(n-1)!}{2}$

Derangements :

If n distinct items are arranged, the number of ways they can be arranged so that they do not occupy their intended spot is

$$D = n! \left(\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!} \right)$$

Tip 1 – S.I and C.I

- Simple Interest (S.I) and Compound Interest (C.I) is one of the easiest topics in quantitative section.
- Every year, a significant number of questions appear from each of these sections and students should aim to get all the questions right from these topics.
- The number of concepts in these topics is limited and most of the problems can be solved by applying the formulae directly.
- Many students commit silly mistakes in this topic due to complacency and this should be avoided.

Tip 2 – S.I & C.I

- In Simple Interest the principal and the Interest (occurred every period) remains constant
- In Compound Interest the Interest earned over the period is added over to the existing principal after every compounding period. So the principal and the Interest over a period changes after every compounding period.
- For the same principal, positive rate of interest and time period (>1 year), the compound interest on the loan is always greater than the simple interest.

Tip 3 – S.I

- The sum of principal and the interest is called Amount.

$$\text{Amount (A)} = \text{Principal (P)} + \text{Interest (I)}$$

- The Simple Interest (I) occurred over a time period (T) for R% (rate of Interest per annum),

$$I = \frac{PTR}{100}$$

Tip 4 – C.I

- The amount to be paid, if money is borrowed at Compound Interest for N number of years,

$$A = P \left(1 + \frac{R}{100} \right)^N$$

- The Interest occurred, $I = A - P$

$$I = P \left(1 + \frac{R}{100} \right)^N - P$$

Tip 5 – C.I

If R is rate of interest per year, N is number of years, P is the principal

- If interest is compounded half yearly, then Amount,

$$A = P \left(1 + \frac{R/2}{100} \right)^{2N}$$

- If interest is compounded quarterly, then Amount,

$$A = \left(1 + \frac{R/4}{100} \right)^{4N}$$

Tip 6 – S.I & C.I

- If interest Rate is $R_1\%$ for first year, $R_2\%$ for second year and $R_3\%$ for 3rd year,

then the Amount, $A = P \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) \left(1 + \frac{R_3}{100}\right)$

- If a difference between C.I and S.I for certain sum at same rate of interest is given, then Principal = Difference $(100/R)^2$

- When interest is compounded annually but time is in fraction, let $a\frac{b}{c}$ then

the Amount, $A = P \left(1 + \frac{R}{100}\right)^a \left(1 + \frac{R \frac{b}{c}}{100}\right)$

Tip 7 – S.I & C.I

If R is the rate per annum, then present worth of Rs. K due to N years hence is given by

$$\text{Present worth} = \frac{K}{\left(1 + \frac{R}{100}\right)^N}$$

Tip 7 – Quadratic Equations

If $A_n X^n + A_{n-1} X^{n-1} + \dots + A_1 X + A = 0$, then

- Sum of the roots $= -A_{n-1}/A_n$
- Sum of roots taken two at a time $= A_{n-2}/A_n$
- Sum of roots taken three at a time $= -A_{n-3}/A_n$ and so on
- Product of the roots $= [(-1)^n A_0] / A_n$

Tip 1 – Profit, Loss and Discount

- Profit, Loss and Discount is very important topic for CAT and significant number of questions are asked from this topic every year.
- The number of concepts in these topics is limited and most of the problems can be solved by applying the formulae directly
- This document covers various formulas, tips and shortcuts of Profit, Loss and Discount topic.

Tip 2 – Profit, Loss and Discount

- **Cost Price**

The amount paid to purchase an article or the cost of manufacturing an article is called Cost Price (C.P)

- **Selling Price**

The price at which a product is sold is called Selling price (S.P)

- **Marked Price**

The price at which an article is marked is called Marked price (M.P)

Tip 3 – Profit, Loss and Discount

- If $S.P > C.P$, then Profit or Gain, $P = S.P - C.P$
- If $C.P > S.P$, then Loss, $L = C.P - S.P$
- % Profit or Gain percentage or Profit Percentage = $\frac{\text{Profit}}{C.P} \times 100$
- %Loss = $\frac{\text{Loss}}{C.P} \times 100$
- Discount = $M.P - S.P$ (If no discount is given, then $M.P = S.P$)
- %Discount = $\frac{\text{Discount}}{M.P} \times 100$

Tip 4 – Profit, Loss and Discount

- Total increase in price due to two subsequent increases of X% and Y% is $(X + Y + \frac{XY}{100})\%$
- If two items are sold at same price, each at Rs. x, one at a profit of P% and other at a loss of P% then there will be overall loss of $\frac{P^2}{100}\%$

$$\text{The absolute value of loss} = \frac{2P^2x}{100^2 - P^2}$$

Tip 5 – Profit, Loss and Discount

- If C.P of two items is same, and by selling of each item he earned p% profit on one article and p% loss on another, then there will be no loss or gain.
- If a trader professes to sell at C.P but uses false weight, then

$$\text{Gain\%} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100$$

Tip 6 – Profit, Loss and Discount

- $S.P = \left(\frac{100 + \text{Profit}\%}{100} \right) C.P$ (If $S.P > C.P$)
- $S.P = \left(\frac{100 - \text{Loss}\%}{100} \right) C.P$ (If $S.P < C.P$)
- $C.P = \frac{100 \times S.P}{100 + \text{Profit}\%}$ (If $S.P > C.P$)
- $C.P = \frac{100 \times S.P}{100 - \text{Loss}\%}$ (If $S.P < C.P$)

Tip 7 – Profit, Loss and Discount

- Buy x get y free, then the %discount = $\frac{y}{x+y} \times 100$.
(here x+y articles are sold at C.P of x articles.)

- When there are two successive discounts of a% and b% are given then the,

$$\text{Resultant discount} = \left(a + b - \frac{a*b}{100} \right)$$

- If C.P of x article is equal to the selling price of y articles then the,

$$\text{Resultant profit \% or loss \%} = \frac{y-x}{y} \times 100$$

Tip 6 – Quadratic Equations

- Minimum and maximum values of $ax^2+bx+c = 0$
- If $a > 0$: minimum value = $\frac{4ac-b^2}{4a}$ and occurs at $x = -b/2a$
- If $a < 0$: maximum value = $\frac{4ac-b^2}{4a}$ and occurs at $x = -$

Tip 1 – Sets and Venn diagrams

- Its one of the easiest topics of CAT.
- Most of the formulae in this section can be deduced logically with little effort.
- The difficult part of the problem is translating the sentences into areas of the Venn diagram.
- While solving, pay careful attention to phrases like and, or, not, only, in as these generally signify the relationship.

Tip 2 – Sets and Venn diagrams

- Set is defined as a collection of well-defined objects.
Ex. Set of whole numbers
- Every object is called Element of the set.
- The number of elements in the set is called cardinal number

Tip 3 – Sets and Venn diagrams

Types of Sets

1. Null set:

A set with zero or no elements is called Null set. It is denoted by $\{ \}$ or $\emptyset$. Null set cardinal number is 0

2. Singleton set:

Sets with only one element in them are called singleton sets.

Ex. $\{2\}$, $\{a\}$, $\{0\}$

3. Finite and Infinite set:

A set having finite number of elements is called finite set. A set having infinite or

Tip 4 – Sets and Venn diagrams

Types of Sets

4. Universal set:

A set which contains all the elements of all the sets and all the other sets in it, is called universal set.

5. Subset:

A set is said to be subset of another set if all the elements contained in it are also part of another set. Ex. If $A = \{1,2\}$, $B = \{1,2,3,4\}$ then, Set “A” is said to be subset of set B.

Tip 5 – Sets and Venn diagrams

Types of Sets

6. Equal sets:

Two sets are said to be equal sets when they contain same elements

Ex. $A = \{a,b,c\}$ and $B = \{a,b,c\}$ then A and B are called equal sets.

7. Disjoint sets:

When two sets have no elements in common then the two sets are called disjoint sets

Ex. $A = \{1,2,3\}$ and $B = \{6,8,9\}$ then A and B are disjoint sets.

Tip 6 – Sets and Venn diagrams

Types of Sets

8. Power set:

- A power set is defined as the collection of all the subsets of a set and is denoted by $P(A)$
- If $A = \{a, b\}$ then $P(A) = \{ \{ \}, \{a\}, \{b\}, \{a, b\} \}$
- For a set having n elements, the number of subsets are 2^n

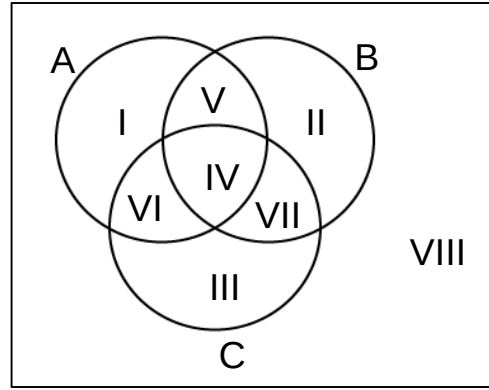
Tip 7 – Sets and Venn diagrams

Properties of Sets:

- The null set is a subset of all sets
- Every set is subset of itself
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup \emptyset = A$

Tip 8 – Sets and Venn diagrams

Venn diagrams: A Venn diagram is a figure to represent various sets and their relationship.



I, II, III are the elements in only A, only B and only C respectively

IV – Elements which are in all of A, B and C.

V - Elements which are in A and B but not in C.

VI – Elements which are in A and C but not in B.

VII – Elements which are in B and C but not in A.

VIII – Elements which are not in either A or B or C.

Tip 9 – Sets and Venn diagrams

Union of sets is defined as the collection of elements either in A or B or both. It is represented by symbol “U”. Intersection of set is the collection of elements which are in both A and B.

- Let there are two sets A and B then,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

- If there are 3 sets A, B and C then,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Tip 10 – Sets and Venn diagrams

To maximize overlap,

- Union should be as small as possible
- Calculate the surplus = $n(A) + n(B) + n(C) - n(A \cup B \cup C)$
- This can be attributed to $n(A \cap B \cap C')$, $n(A \cap B' \cap C)$, $n(A' \cap B \cap C)$, $n(A \cap B \cap C)$.
- To maximize the overlap, set the other three terms to zero.

Tip 11 – Sets and Venn diagrams

To minimize overlap,

- Union should be as large as possible
- Calculate the surplus = $n(A) + n(B) + n(C) - n(A \cup B \cup C)$
- This can be attributed to $n(A \cap B \cap C')$, $n(A \cap B' \cap C)$, $n(A' \cap B \cap C)$, $n(A \cap B \cap C)$.
- To minimize the overlap, set the other three terms to maximum possible.

Tip 12 – Sets and Venn diagrams

Some other important properties

- A' is called complement of set A , or $A' = U - A$
- $n(A - B) = n(A) - n(A \cap B)$
- $A - B = A \cap B'$
- $B - A = A' \cap B$
- $(A - B) \cup B = A \cup B$

Tip 1 – Mixtures and Alligations

- The topic mixtures and alligations is basically an application of averages concept in CAT.
- The theory involved in this topic is very limited and students should be comfortable with the some basic formulas and concepts.
- This pdf covers all the important formulas and concepts related to mixtures and alligations.

Tip 2 – Mixtures and Alligations

A mixture is created when two or more substances are mixed in a certain ratio.

Types of mixtures

1. Simple mixture

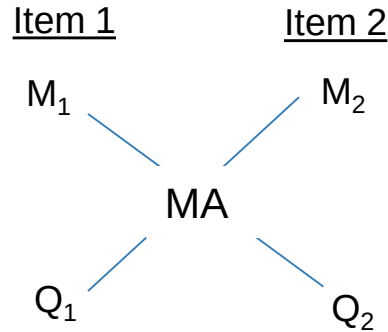
A simple mixture is formed by the mixture of two or more different substances.

Ex. Water and Wine mixture

2. Compound mixture

Compound mixture is formed by the mixture of two or more simple mixtures.

Tip 3 – Mixtures and Alligations



If M_1 and M_2 are the values, Q_1 and Q_2 are the quantities of item 1 and item 2 respectively and M_A is the weighted average of the two items, then

$$\frac{Q_1}{Q_2} = \frac{M_2 - M_A}{M_A - M_1}$$

Weighted average M_A can be calculated by, $M_A = \frac{Q_1 M_1 + Q_2 M_2}{Q_1 + Q_2}$

Tip 4 – Mixtures and Alligations

The alligation rule can be applied when cheaper substance is mixed with expensive substance

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{Price of dearer} - \text{Mean price}}{\text{Mean price} - \text{Price of cheaper}}$$

Tip 5 – Mixtures and Alligations

If two mixtures M_1 and M_2 , having substances S_1 and S_2 in the ratio $a:b$ and $p:q$ respectively are mixed, then in the final mixture,

$$\frac{\text{Quantity of } S_1}{\text{Quantity of } S_2} = \frac{M_1\left[\frac{a}{a+b}\right] + M_2\left[\frac{p}{p+q}\right]}{M_1\left[\frac{b}{a+b}\right] + M_2\left[\frac{q}{p+q}\right]}$$

Tip 6 – Mixtures and Alligations

If there is a container with 'a' liters of liquid A and if 'b' liters are withdrawn and equal amount is replaced by another liquid B and if the operation is repeated for 'n' times

After nth operation,

- Liquid A in the container = $\left[\frac{a-b}{a}\right]^n \times \text{Initial quantity of A in the container}$
- $\frac{\text{Liquid A after nth operation}}{\text{Liquid B after nth operation}} = \frac{\left[\frac{a-b}{a}\right]^n}{1 - \left[\frac{a-b}{a}\right]^n}$

Tip 1 – Progressions and Series

- Progressions and Series is one of the important topics for CAT and significant number of questions appear in the examination from this section every year.
- of the questions from this section can be very tough and time consuming while the others can be very easy.
- The trick to ace this section is to quickly figure out whether a question is solvable or not and not waste time on very difficult questions.

- Some of the questions in this section can be answered by ruling out wrong choices among the options available. This method will both save time and improve accuracy.
- There are many shortcuts which will be of vital importance in answering this section.
- This formula sheet contains an exhaustive list of various formulas and shortcuts.

Tip 3 – Progressions and Series

There are 3 standard types of progressions

- Arithmetic Progression
- Geometric Progression
- Harmonic Progression

Tip 4 – Progressions and Series

Arithmetic progression (A.P)

- If the sum or difference between any two consecutive terms is constant then the terms are said to be in A.P
- Ex. 2,5,8,11 or $a, a+d, a+2d, a+3d\dots$
- If 'a' is the first term and 'd' is the difference then the general 'n' term is $T_n = a + (n-1)d$
- Sum of first 'n' terms in A.P $= \frac{n}{2} [2a+(n-1)d]$
- Number of terms in A.P $= \frac{\text{Last term} - \text{First term}}{\text{Common difference}} + 1$

Tip 5 – Progressions and Series

Properties of A.P

If $a, b, c, d, \dots$ are in A.P and 'k' is a constant then

- $a-k, b-k, c-k, \dots$ will also be in A.P
- $ak, bk, ck, \dots$ will also be in A.P
- $a/k, b/k, c/k$ also be in A.P

Tip 6 – Progressions and Series

Geometric Progression

- If in a succession of numbers the ratio of any term and the previous term is constant then that numbers are said to be in Geometric Progression.
- Ex :1, 3, 9, 27 or a, ar, ar^2, ar^3
- The general expression of an G.P, $T_n = ar^{n-1}$ (where a is the first terms and 'r' is the common ratio)
- Sum of 'n' terms in G.P, $S_n = \frac{a(1-rn)}{1-r}$ (If $r < 1$) or $\frac{a(rn-1)}{r-1}$ (If $r > 1$)

Tip 7 – Progressions and Series

Properties of G.P

If $a, b, c, d, \dots$ are in G.P and 'k' is a constant then

1. $ak, bk, ck, \dots$ will also be in G.P
2. $a/k, b/k, c/k$ will also be in G.P

Sum of term of infinite series in G.P, $S_{\infty} = \frac{a}{1-r}$ (-

Tip 8 – Progressions and Series

Harmonic Progression

- If $a, b, c, d, \dots$ are unequal numbers then they are said to be in H.P if $1/a, 1/b, 1/c, \dots$ are in A.P
- The 'n' term in H.P is $1/(\text{nth term in A.P})$

Properties of H.P :

If $a, b, c, d, \dots$ are in H.P, then

$$a+d > b+c$$

$$ad > bc$$

Tip 9 – Progressions and Series

Arithmetic Geometric Series

- A series will be in arithmetic geometric series if each of its term is formed by product of the corresponding terms of an A.P and G.P.
- The general form of A.G.P series is $a, (a+d)r, (a+2d)r^2, \dots$
- Sum of 'n' terms of A.G.P series

$$S_n = \frac{a}{1-r} + rd \frac{(1-r^{n-1})}{1-r} + rn \frac{[a+(n-1)d]}{1-r} \quad (r \neq 1)$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Tip 10 – Progressions and Series

Arithmetic Geometric Series

- Sum of infinite terms of A.G.P series

$$S_{\infty} = \frac{a}{1-r} + rd \frac{1}{(1-r)^2} \quad (r < 1)$$

Tip 11 – Progressions and Series

Standard Series

- The sum of first 'n' natural number = $\frac{n(n+1)}{2}$
- The sum of squares of first 'n' natural numbers = $\frac{n(n+1)(2n+1)}{6}$
- The sum of cubes of first 'n' natural numbers = $\left(\frac{n(n+1)}{2}\right)^2$
- $\frac{n(n+1)(2n+1)}{6}$
- The sum of first 'n' even natural numbers = $n(n+1)$
- In any series $T_n = S_n - S_{n-1}$

Tip 12 – Progressions and Series

Arithmetic mean

- The arithmetic mean = $\frac{\text{Sum of all the terms}}{\text{Number of Terms}}$
- If two number A and B are in A.P then arithmetic mean = $\frac{a+b}{2}$

Arithmetic mean

- Inserting 'n' means between two numbers a and b
- The total terms will become n+2, a is the first term and b is the last term
- Then the common difference $d = \frac{b-a}{n+1}$
- The last term $b = a+(n+1)d$
- The final series is a, a+d, a+2d,....

Tip 14 – Progressions and Series

Geometric Mean

- If $a, b, c, \dots$ n terms are in G.P then $G.M = \sqrt[n]{a \times b \times c \times \dots n \text{ terms}}$
- If two numbers a, b are in G.P then their $G.M = \sqrt{a \times b}$

Tip 15 – Progressions and Series

Geometric Mean

- Inserting 'n' means between two quantities a and b with common ratio 'r'
- Then the number of terms are n+2 and a, b are the first and last terms
- $r^{n+1} = \frac{b}{a}$ or $r = \frac{\sqrt[n+1]{b}}{a}$
- The final series is a, ar, ar²,...

Tip 16 – Progressions and Series

Harmonic Mean

- If $a, b, c, d, \dots$ are the given numbers in H.P then the Harmonic mean of 'n' terms =
$$\frac{\text{Number of terms}}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \dots}$$
- If two numbers a and b are in H.P then the Harmonic mean =
$$\frac{2ab}{a+b}$$

Tip 17 – Progressions and Series

Relationship between AM, GM and HM for two numbers a and b,

- $AM = \frac{a+b}{2}$
- $G.M = \sqrt{a \times b}$
- $H.M = \frac{2ab}{a+b}$
- $G.M = \sqrt{AM \times HM}$
- $A.M \geq G.M \geq H.M$

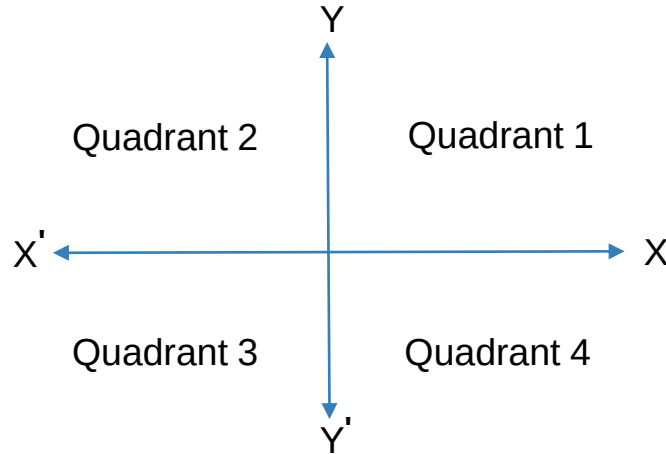
Tip 1 - Geometry

- Geometry is one of the hardest sections to crack without preparation and one of the easiest with preparation.
- With so many formulas to learn and remember, this section is going to take a lot of time to master.
- Remember, read a formula, try to visualize the formula and solve as many questions related to the formula as you can.

Tip 2 - Geometry

- Knowing a formula and knowing when to apply it are two different abilities.
- The first will come through reading the formulae list and theory but the latter can come only through solving many different problems.
- So in this document we are going to provide an exhaustive list of formulas and tips for making geometry section a lot easier.
- Try to remember all of them and don't forget to share.

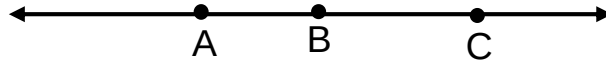
Tip 3 - Geometry



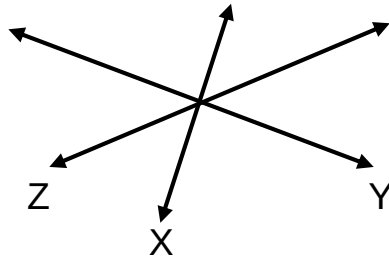
Quadrant I	X is Positive	Y is Positive
Quadrant II	X is Negative	Y is Positive
Quadrant III	X is Negative	Y is Negative
Quadrant IV	X is Positive	Y is Negative

Tip 4 - Geometry

- Collinear points : Three or more points lying on the single straight line. In this diagram the three points A,B and C are collinear



- Concurrent lines : If three or more lines lying in a same plane intersect at a single point then that lines are called concurrent lines. The three lines X, Y and Z are here concurrent lines.



Tip 5 - Geometry

- The distance between two points with coordinates (X_1, Y_1) , (X_2, Y_2) is given by $D = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$
- Slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$ (If $x_2 = x_1$ then the lines are perpendicular to each other)
- Mid point between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$
- When two lines are parallel, their slopes are equal i.e. $m_1 = m_2$
- -1 i.e. $m_1 * m_2 = -1$

Tip 6 - Geometry

- If two intersecting lines have slopes m_1 and m_2 then the angle between two lines will be $\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$ (where θ is the angle between the lines)

- The length of perpendicular from a point (X_1, Y_1) on the line $AX + BY + C = 0$ is

$$P = \frac{AX_1 + BY_1 + C}{\sqrt{A^2 + B^2}}$$

- The distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is

$$\left| \frac{C_1 - C_2}{\sqrt{A^2 + B^2}} \right|$$

Tip 7 - Geometry

Equations of a lines :

General equation of a line	$Ax + By = C$
Slope intercept form	$y = mx + c$ (c is y intercept)
Point-slope form	$y - y_1 = m (x - x_1)$
Intercept form	$\frac{x}{a} + \frac{y}{b} = 1$
Two point form	$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

Tip 8 - Geometry

General equation of a circle :

The general equation of a circle is $x^2+y^2+2gx+2fy+c=0$

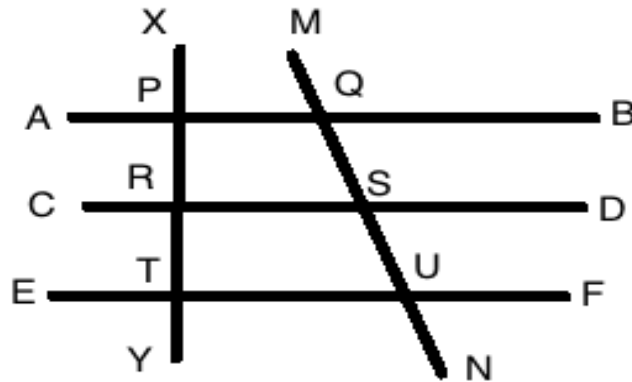
- Centre of the circle is $(-g,-f)$
- Radius of the circle $= \sqrt{g^2 + f^2 - c}$
- If the origin is the centre of the circle then equation of the circle is $x^2+y^2=r^2$

Tip 9 - Geometry

- When two angles A and B are complementary, sum of A and B is 90°
- When two angles A and B are supplementary, sum of A and B is 180°
- When two lines intersect, opposite angles are equal. Adjacent angles are supplementary
- When any number of lines intersect at a point, the sum of all the angles formed = 360°

Tip 10 - Geometry

- Consider parallel lines AB, CD and EF as shown in the figure.



- XY and MN are known as transversals
- $\angle XPQ = \angle PRS = \angle RTU$ as corresponding angles are equal

Tip 11 - Geometry

- Interior angles on the side of the transversal are supplementary.
i.e. $\angle PQS + \angle QSR = 180^\circ$
- Exterior angles on the same side of the transversal are supplementary. i.e. $\angle MQB + \angle DSU = 180^\circ$
- Two transversals are cut by three parallel lines in the same ratio i.e.
$$\frac{PR}{RT} = \frac{QS}{SU}$$
- Interior angles on the side of the transversal are supplementary.
i.e. $\angle PQS + \angle QSR = 180^\circ$
- Exterior angles on the same side of the transversal are supplementary.
i.e. $\angle MQB + \angle DSU = 180^\circ$
- Two transversals are cut by three parallel lines in the same ratio i.e. $\frac{PR}{RT} = \frac{QS}{SU}$

Tip 12 - Geometry

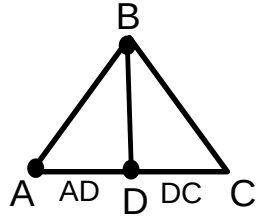
- Sum of all angles in a triangle is 180°
- An angle less than 90° is called an acute angle. An angle greater than 90° is called an obtuse angle.
- A triangle with all sides unequal is called scalene triangle
- A triangle with two sides equal is called isosceles triangle. The two angles of an isosceles triangle that are not contained between the equal sides are equal
- A triangle with all sides equal is called equilateral triangle. All angles of an equilateral triangle equal 60° .

Tip 13 - Geometry

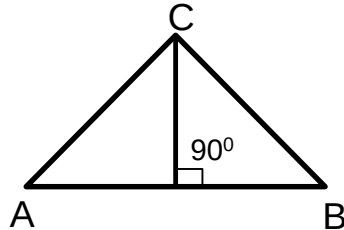
- If in a triangle all of its angles are less than 90° then that triangle is called as acute angled triangle
- A triangle with one of its angle equal to 90° then that triangle is called as Right angled triangle
- A triangle with one of its angle greater than 90° then that triangle is called as Obtuse angled triangle
- If one side of a triangle is produced then that exterior angle formed is equal to the sum of opposite remote interior angles

Tip 14 - Geometry

- A line joining the mid point of a side with the opposite vertex is called a median. (Here D is the midpoint of AC side or $AD = DC$). BD is the median of this triangle ABC.



- A perpendicular drawn from a vertex to the opposite side is called the altitude



Tip 15 - Geometry

- A line that bisects and also makes right angle with the same side of the triangle is called perpendicular bisector
- A line that divides the angle at one of the vertices into two parts is called angular bisector
- All points on an angular bisector are equidistant from both arms of the angle.
- All points on a perpendicular bisector of a line are equidistant from both ends of the line.
- In an equilateral triangle, the perpendicular bisector, median, angle bisector and altitude (drawn from a vertex to a side) coincide.

Tip 16 - Geometry

- The point of intersection of the three altitudes is the Orthocentre
- The point of intersection of the three medians is the centroid.
- The three perpendicular bisectors of a triangle meet at a point called the Circumcentre. A circle drawn from this point with the circumradius would pass through all the vertices of the triangle.
- The three angle bisectors of a triangle meet at a point called the incentre of a triangle. The incentre is equidistant from the three sides and a circle drawn from this point with the inradius would touch all the sides of the triangle.

Tip 17 - Geometry

- Sum of any two sides of a triangle is always greater than it's third side
- Difference of any two sides of a triangle is always lesser than it's third side

Pythagoras theorem :

In a right angled triangle ABC where $\angle B = 90^\circ$, $AC^2 = AB^2 + BC^2$

Apollonius theorem

In a triangle ABC, if AD is the median to side BC then by Apollonius theorem,

$$2*(AD^2 + BD^2) = AC^2 + AB^2$$

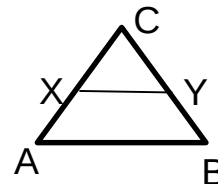
Tip 18 - Geometry

Mid Point Theorem :

The line joining the midpoint of any two sides in a triangle is parallel to the third side and is half the length of the third side.

If X is the midpoint of CA and Y is the midpoint of CB

Then XY will be parallel to AB and $XY = \frac{1}{2} * AB$

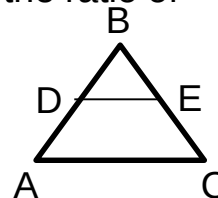


Basic proportionality theorem :

If a line is drawn parallel to one side of a triangle and it intersects the other two sides at two distinct points then it divides the two sides in the ratio of respective sides

If in a triangle ABC, D and E are the points lying on AB and BC respectively and DE is parallel to AC then

$AD/DB = EC/BE$

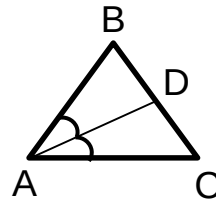


Tip 19 - Geometry

Interior Angular Bisector theorem :

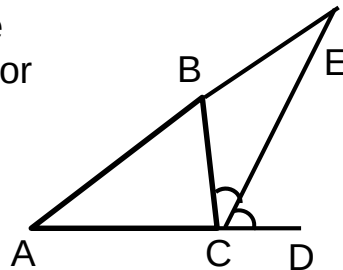
In a triangle the angular bisector of an angle divides the side opposite to the angle, in the ratio of the remaining two sides. In a triangle ABC if AD is the angle bisector of angle A then AD divides the side BC in the same ratio as the other two sides of the triangle.

$$\text{i.e. } BD/CD = AB/AC.$$



Exterior Angular Bisector theorem :

The angular bisector of exterior angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle. In a triangle ABC, if CE is the angular bisector of exterior angle BCD of a triangle, then $AE/BE = AC/BC$

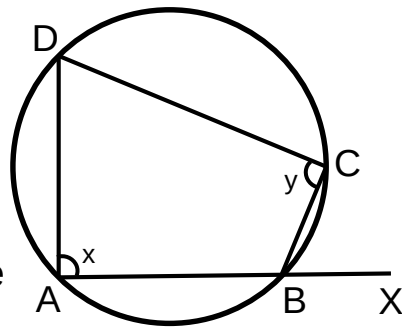


Tip 20 - Geometry

Cyclic Quadrilateral :

If a quadrilateral has all its vertices on the circle and its opposite angles are supplementary (here $x+y = 180^\circ$) then that quadrilateral is called cyclic quadrilateral.

- In a cyclic quadrilateral the opposite angles are supplementary.



- Area of a cyclic quadrilateral is $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ where $s = (a+b+c+d)/2$

-

$$\angle CBX = \angle ADC$$

Tip 21 - Geometry

- If x is the side of an equilateral triangle then the

$$\text{Altitude (h)} = \frac{\sqrt{3}}{2} x$$

$$\text{Area} = \frac{\sqrt{3}}{4} x^2$$

$$\text{Inradius} = \frac{1}{3} * h$$

$$\frac{2}{3} * h$$

- Area of an isosceles triangle = $\frac{a}{4} \sqrt{4c^2 - a^2}$

(where a, b and c are the length of the sides of BC, AC and AB respectively and b = c)

Tip 22 - Geometry

Similar triangles :

If two triangles are similar then their corresponding angles are equal and the corresponding sides will be in proportion.

For any two similar triangles :

- Ratio of sides = Ratio of medians = Ratio of heights = Ratio of circumradii = Ratio of Angular bisectors
- Ratio of areas = Ratio of the square of the sides.

Tests of similarity : (AA / SSS / SAS)

Tip 23 - Geometry

Congruent triangles

If two triangles are congruent then their corresponding angles and their corresponding sides are equal.

Tests of congruence : (SSS / SAS / AAS / ASA)

Tip 24 - Geometry

Area of a triangle, A :

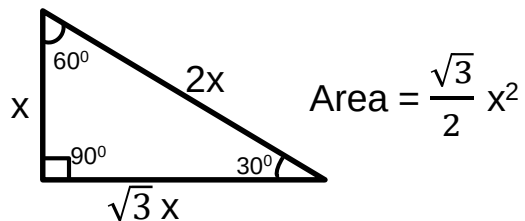
- $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = (a+b+c)/2$.
- $A = 1/2 * \text{base} * \text{altitude}$
- $A = 1/2 * ab * \sin C$ (C is the angle formed between sides a and b)
- $A = \frac{abc}{4R}$ where R is the circumradius
- $A = r * s$ where r is the inradius and s is the semi perimeter.

(where a, b and c are the lengths of the sides BC, AC and AB)

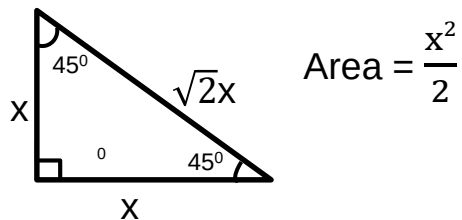
Tip 25 - Geometry

Special triangles :

30° , 60° and 90°

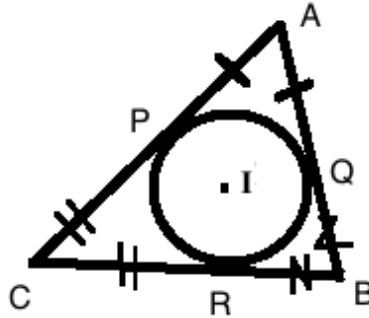


45° , 45° , 90°



Tip 26 - Geometry

- Consider the triangle ABC with incentre I, and the incircle touching the triangle at P,Q,R as shown in the diagram. As tangents drawn from a point are equal, $AP=AQ$, $CP=CR$ and $BQ=BR$.

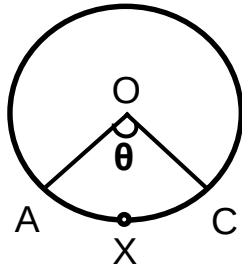


- If a is the side of an equilateral triangle, circumradius = $a/\sqrt{3}$ and inradius = $a/(2\sqrt{3})$

Tip 27 - Geometry

- The angle subtended by a diameter of circle on the circle = 90°
- Angles subtended by a equal chord are equal. Also, angles subtended in the major segment are half the angle formed by the chord at the centre
- Equal chords of a circle or equidistant from the centre
- The radius from the centre to the point where a tangent touches a circle is perpendicular to the tangent
- Tangents drawn from the same point to a circle are equal in length
- A perpendicular drawn from the centre to any chord, bisects the chord

Tip 28 - Geometry

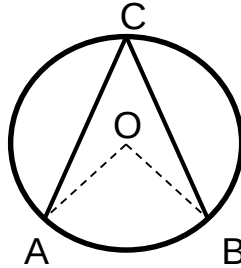


$$\text{Area of sector OAXC} = \frac{\theta}{360} * \pi r^2$$

$$\text{Area of minor segment AXC} = \frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \sin \theta$$

Tip 29 - Geometry

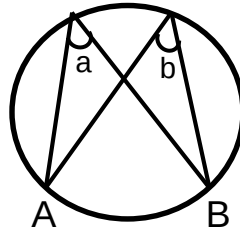
Inscribed angle Theorem :



$$2\angle ACB = \angle AOB$$

The angle inscribed by the two points lying on the circle, at the centre of the circle is twice the angle inscribed at any point on the circle by the same points.

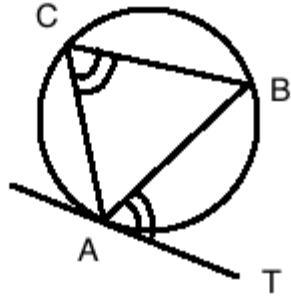
Tip 30 - Geometry



Angles subtended by the same segment on the circle will be equal. So here angles a and b will be equal

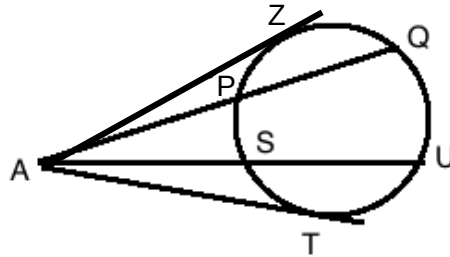
Tip 31 - Geometry

The angle made by a chord with a tangent to one of the ends of the chord is equal to the angle subtended by the chord in the other segment. As shown in the figure, $\angle ACB = \angle BAT$.



Tip 32 - Geometry

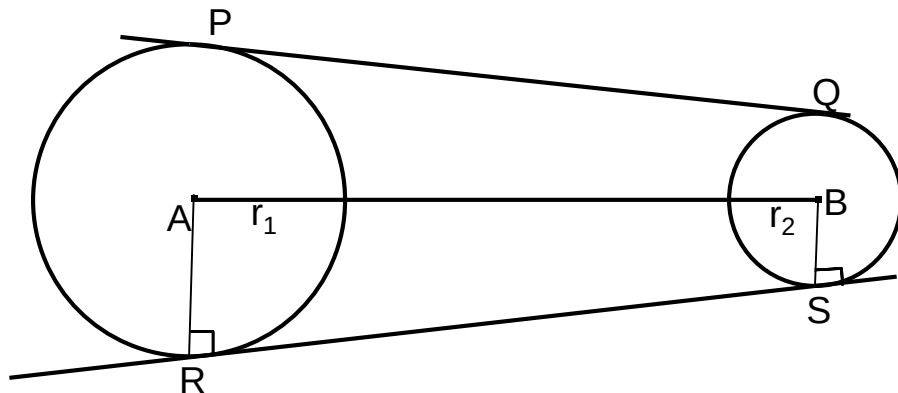
Consider a circle as shown in the image. Here, $AP \cdot AQ = AS \cdot AU = AT^2$



Two tangents drawn to a circle from an external common point will be equal in length. So here $AZ = AT$

Tip 33 - Geometry

Direct common tangent :

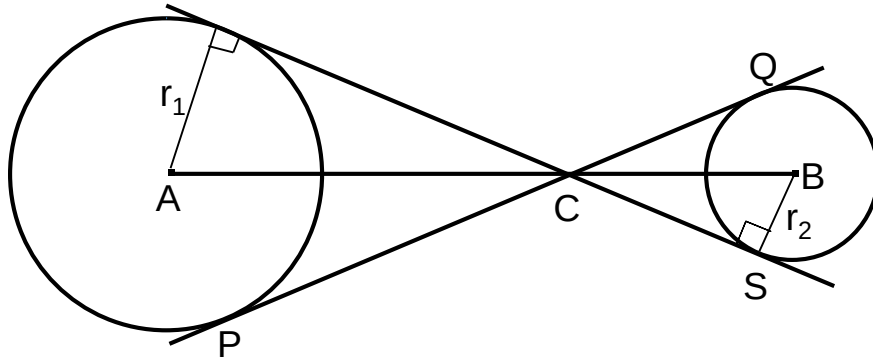


In this figure PQ and RS are the direct common tangents and let AB
(Distance between the two centres) = D

$$PQ^2 = RS^2 = D^2 - (r_1 - r_2)^2$$

Tip 34 - Geometry

Transverse common tangent :



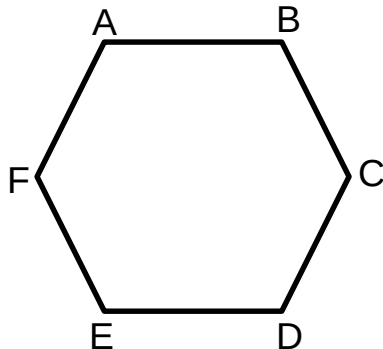
In this figure PQ and RS are the transverse common tangents and let AB (Distance between the two centres) = D

$$PQ^2 = RS^2 = D^2 - (r_1 + r_2)^2$$

Tip 35 - Geometry

- If all sides and all angles are equal, then the polygon is a regular polygon
- A regular polygon of n sides has $n(n-3)/2$ diagonals
- In a regular polygon of n sides, each exterior angle is $360/n$ degrees.
- Sum of measure of all the interior angles of a regular polygon is $180(n-2)$ degrees (where n is the number of sides of the polygon)
- Sum of measure of all the exterior angles of regular polygon is 360 degrees

Tip 36 - Geometry



ABCDEF is a regular hexagon with each side equal to 'x' then

- Each interior angle = 120°
- Each exterior angle = 60°
- Sum of all the exterior angles = 360°
- Sum of all the interior angles = 720°
- Area = $\frac{3\sqrt{3}}{2} a^2$

Tip 37 - Geometry

Areas of different geometrical figures

Triangle	$\frac{1}{2} * \text{base} * \text{height}$
Rectangle	$\text{length} * \text{width}$
Trapezoid	$\frac{1}{2} * \text{sum of bases} * \text{height}$
Parallelogram	$\text{base} * \text{height}$
Circle	$\pi * \text{radius}^2$
Rhombus	$\frac{1}{2} * \text{product of diagonals}$
square	$\text{side}^2 \text{ or } \frac{1}{2} \text{ diagonal}^2$
Kite	$\frac{1}{2} * \text{product of the diagonals}$

Tip 38 - Geometry

Volume of different solids

Cube	length^3
Cuboid	$\text{Length} * \text{base} * \text{height}$
Prism	$\text{Area of base} * \text{height}$
Cylinder	$\pi r^2 h$
Pyramid	$1/3 * \text{Area of base} * \text{Height}$
Cone	$1/3 * \pi r^2 * h$
Cone Frustum (If R is the base radius, r is the upper surface radius and h is the height of the frustum)	$1/3 \pi h (R^2 + Rr + r^2)$
	$4/3 * \pi * r^3$
Hemi-sphere	$2/3 \pi r^3$

Tip 39 - Geometry

Total Surface area of different solids

Prism	$2 \times \text{base area} + \text{base perimeter} \times \text{height}$
Cube	$6 \times \text{length}^2$
Cuboid	$2(lh + bh + lb)$
Cylinder	$2\pi rh + 2\pi r^2$
Pyramid	$\frac{1}{2} \times \text{Perimeter of base} \times \text{slant height} + \text{Area of base}$
Cone (l is the slant height)	$\pi r(l + r)$
Cone Frustum (where R and r are the radii of the base faces and l is the slant height)	$\pi(R^2 + r^2 + Rl + rl)$
Sphere	$4\pi r^2$
Hemi-sphere	$3\pi r^2$

Tip 40 - Geometry

Lateral/Curved surface area

Prism	$\text{base perimeter} \times \text{height}$
Cube	$4 \times \text{length}^2$
Cuboid	$2\text{length} \times \text{height} + 2 \times \text{breadth} \times \text{height}$
Cylinder	$2\pi rh$
Pyramid	$\frac{1}{2} \times \text{Perimeter of base} \times \text{slant height}$
Cone (l is the slant height)	πrl
Cone Frustum (where R and r are the radii of the base faces and l is the slant height)	π

Tip 41 - Geometry

- The angle subtended by a diameter of circle on the circle = 90 degrees
- Angles subtended by a equal chords are equal. Also, angles subtended in the major segment are half the angle formed by the chord at the centre
- The radius from the centre to the point where a tangent touches a circle is perpendicular to the tangent
- Tangents drawn from the same point to a circle are equal in length
- A perpendicular drawn from the centre to any chord, bisects the chord
- In a regular polygon of n sides, each exterior angle is $360/n$ degrees.

Time, Speed, Distance and Work Tips

- Time, Distance and Work is the most important topic for all Competitive examinations.
- The questions from this topic varies from easy to difficult.
- This formula sheet covers the most importance tips that helps you to answer the questions in a easy, fast and accurate way.

Tip 1 – Time, Speed, Distance & Work

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Tip 2 – Time, Speed, Distance & Work

If the ratio of the speeds of A and B is $a : b$, then

- The ratio of the times taken to cover the same distance is $1/a : 1/b$ or $b : a$.
- The ratio of distance travelled in equal time intervals is $a : b$

Tip 3 – Time, Speed, Distance & Work

- Average speed = $\frac{\text{Total Distance travelled}}{\text{Total Time taken}}$
- If a part of a journey is travelled at speed S_1 km/hr in T_1 hours and remaining part at speed S_2 km/hr in T_2 hours then,

$$\text{Total distance travelled} = S_1T_1 + S_2T_2 \text{ km}$$

$$\text{Average speed} = \frac{S_1T_1 + S_2T_2}{T_1 + T_2} \text{ km/hr}$$

Tip 4 – Time, Speed, Distance & Work

- In a journey travelled with different speeds, if the distance covered in each stage is constant, the average speed is the harmonic mean of the different speeds.
- Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr

Then the average speed during the whole journey is $\frac{2xy}{x+y}$ km/hr

Tip 5 – Time, Speed, Distance & Work

- In a journey travelled with different speeds, if the time travelled in each stage is constant, the average speed is the arithmetic mean of the different speeds.
- If a man travelled for certain time at the speed of x km/hr and travelled for equal amount of time at the speed of y km/hr then

The average speed during the whole journey is $\frac{x+y}{2}$ km/hr

Tip 6 – Time, Speed, Distance & Work

Constant distance :

Let the distances travelled in each part of the journey be d_1, d_2, d_3 and so on till d_n and the speeds in each part be s_1, s_2, s_3 and so on till s_n .

If $d_1 = d_2 = d_3 = \dots = d_n = d$, then the average speed is the harmonic mean of the speeds s_1, s_2, s_3 and so on till s_n .

Constant time :

Let the distances travelled in each part of the journey be d_1, d_2, d_3 and so on till d_n and the time taken for each part be t_1, t_2, t_3 and so on till t_n .

If $t_1 = t_2 = t_3 = \dots = t_n = t$, then the average speed is the arithmetic mean of the speeds s_1, s_2, s_3 and so on till s_n .

Tip 7 – Time, Speed, Distance & Work

Circular Tracks :

If two people are running on a circular track with speeds in ratio $a:b$ where a and b are co-prime, then

- They will meet at $a+b$ distinct points if they are running in opposite direction.
- They will meet at $|a-b|$ distinct points if they are running in same direction

Tip 8 – Time, Speed, Distance & Work

Circular Tracks

If two people are running on a circular track having perimeter l , with speeds m and n ,

- The time for their first meeting = $l / (m + n)$
(when they are running in opposite directions)
- The time for their first meeting = $l / (|m - n|)$
(when they are running in the same direction)

Tip 9 – Time, Speed, Distance & Work

If a person P starts from A and heads towards B and another person Q starts from B and heads towards A and they meet after a time 't' then,

$$t = \sqrt{(x*y)}$$

where x = time taken (after meeting) by P to reach B and
y = time taken (after meeting) by Q to reach A.

Tip 10 – Time, Speed, Distance & Work

A and B started at a time towards each other. After crossing each other, they took T_1 hrs, T_2 hrs respectively to reach their destinations. If they travel at constant speeds S_1 and S_2 respectively all over the journey, Then

$$\frac{S_1}{S_2} = \sqrt{\frac{T_2}{T_1}}$$

Tip 11 – Time, Speed, Distance & Work

Trains :

- Two trains of length L_1 and L_2 travelling at speeds of S_1 and S_2 cross each other in a time

$$= \frac{L_1 + L_2}{S_1 + S_2} \text{ (if they are going in opposite directions)}$$

$$= \frac{L_1 + L_2}{|S_1 - S_2|} \text{ (if they are going in the same direction)}$$

Tip 12 – Time, Speed, Distance & Work

Work :

- If X can do a work in 'n' days, the fraction of work X does in a day is $1/n$
- If X can do a work in 'x' days, and Y can do a work in 'y' days,

The number of days taken by both of them together is $\frac{x*y}{x+y}$

- If M_1 men work for H_1 hours per day and worked for D_1 days and completed W_1 work, and if M_2 men work for H_2 hours per day and worked for D_2 days and completed W_2 work, then

$$\frac{M_1 H_1 D_1}{W_1} = \frac{M_2 H_2 D_2}{W_2}$$

Tip 13 – Time, Speed, Distance & Work

Boats and Streams :

If the speed of water is 'W' and speed of a boat in still water is 'B'

- Speed of the boat (downstream) is $B+W$
- Speed of the boat (upstream) is $B-W$

The direction along the stream is called **downstream**.

And, the direction against the stream is called **upstream**.

Tip 14 – Time, Speed, Distance & Work

Boats and Streams :

- If the speed of the boat downstream is x km/hr and the speed of the boat upstream is y km/hr, then

$$\text{Speed of boat in still water} = \frac{x+y}{2} \text{ km/hr}$$

$$\text{Rate of stream} = \frac{x-y}{2} \text{ km/hr}$$

- While converting the speed in m/s to km/hr, multiply it by 3.6 (18/5).
 $1 \text{ m/s} = 3.6 \text{ km/h}$
- While converting km/hr into m/sec, we multiply by 5/18

Tip 15 – Time, Speed, Distance & Work

Pipes and Cisterns :

Inlet Pipe : A pipe which is used to fill the tank is known as Inlet Pipe.

Outlet Pipe : A pipe which can empty the tank is known as Outlet Pipe.

- If a pipe can fill a tank in 'x' hours then the part filled per hour = $1/x$
- If a pipe can empty a tank in 'y' hours, then the part emptied per hour = $1/y$
- If a pipe A can fill a tank in 'x' hours and pipe can empty a tank in 'y' hours, If they are both active at the same time, then

$$\text{The part filled per hour} = \frac{1}{x} - \frac{1}{y} \text{ (If } y > x \text{)}$$

$$\text{The part emptied per hour} = \frac{1}{y} - \frac{1}{x} \text{ (If } x > y \text{)}$$

Tip 16 – Time, Speed, Distance & Work

- Some of the questions may consume a lot of time. While solving, write down the equations as far as possible to avoid mistakes. The few extra seconds can help you avoid silly mistakes.
- Check if the units of distance, speed and time match up. So if you see yourself adding a unit of distance like m to a unit of speed m/s , you would realize you have missed a term.
- Choose to apply the concept of relative speed wherever possible as it can greatly reduce the complexity of the problem.
- Like speed and distance, in time and work while working with terms ensure that you convert all terms to consistent units like man-hours.

Number Systems Tips

- Number Systems is the most important topic in the quantitative section.
- It is a very vast topic and a significant number of questions appear in CAT every year from this section.
- Learning simple tricks like divisibility rules, HCF and LCM, prime number and remainder theorems can help improve the score drastically.
- This document presents best short cuts which makes this topic easy and helps you perform better.

Tip 1 - Number systems

HCF and LCM

- $\text{HCF} * \text{LCM}$ of two numbers = Product of two numbers
- The greatest number dividing a , b and c leaving remainders of x_1 , x_2 and x_3 is the HCF of $(a-x_1)$, $(b-x_2)$ and $(c-x_3)$.
- The greatest number dividing a , b and c ($a < b < c$) leaving the same remainder each time is the HCF of $(c-b)$, $(c-a)$, $(b-a)$.
- If a number, N , is divisible by X and Y and $\text{HCF}(X, Y) = 1$. Then, N is divisible by $X * Y$

Tip 2 - Number systems

Prime and Composite Numbers

- Prime numbers are numbers with only two factors, 1 and the number itself.
- Composite numbers are numbers with more than 2 factors. Examples are 4, 6, 8, 9.
- 0 and 1 are neither composite nor prime.
- There are 25 prime numbers less than 100.

Tip 3 - Number systems

Properties of Prime numbers

- To check if n is a prime number, list all prime factors less than or equal to $\sqrt{n}$. If none of the prime factors can divide n then n is a prime number.
- For any integer a and prime number p , $a^p - a$ is always divisible by p
- All prime numbers greater than 2 and 3 can be written in the form of $6k+1$ or $6k-1$
- If a and b are co-prime then $a^{(b-1)} \bmod b = 1$.

Tip 4 - Number systems

Theorems on Prime numbers

Fermat's Theorem:

Remainder of $a^{(p-1)}$ when divided by p is 1, where p is a prime

Wilson's Theorem:

Remainder when $(p-1)!$ is divided by p is $(p-1)$ where p is a prime

Tip 5 - Number systems

Theorems on Prime numbers

Remainder Theorem

- If a, b, c are the prime factors of N such that $N = a^p * b^q * c^r$
Then the number of numbers less than N and co-prime to N is
$$\phi(N) = N (1 - 1/a) (1 - 1/b) (1 - 1/c).$$

This function is known as the Euler's totient function.

Euler's theorem

- If M and N are co-prime to each other then remainder when $M^{\phi(N)}$ is divided by N is 1.

Tip 6 - Number systems

- Highest power of n in $m!$ is $[m/n] + [m/n^2] + [m/n^3] + \dots$

Ex: Highest power of 7 in $100!$ = $[100/7] + [100/49] = 16$

- To find the number of zeroes in $n!$ find the highest power of 5 in $n!$
- If all possible permutations of n distinct digits are added together the sum = $(n-1)! * (\text{sum of } n \text{ digits}) * (11111... n \text{ times})$

Tip 7 - Number systems

- If the number can be represented as $N = a^p * b^q * c^r \dots$ then number of factors is $(p+1) * (q+1) * (r+1)$
- Sum of the factors = $\frac{a^{p+1} - 1}{a - 1} * \frac{b^{q+1} - 1}{b - 1} * \frac{c^{r+1} - 1}{c - 1}$
-
- If there are n factors, then the number of pairs of factors would be $n/2$. If N is a perfect square then number of pairs (including the square root) is $(n+1)/2$

Tip 8 - Number systems

If the number can be expressed as $N = 2^p * a^q * b^r \dots$ where the power of 2 is p and a, b are prime numbers

- Then the number of even factors of $N = p (1+q) (1+r) \dots$
- The number of odd factors of $N = (1+q) (1+r) \dots$

Tip 9 - Number systems

Number of positive integral solutions of the equation $x^2 - y^2 = k$ is given by

- $\frac{\text{Total number of factors of } k}{2}$ (If k is odd but not a perfect square)
- $\frac{(\text{Total number of factors of } k) - 1}{2}$ (If k is odd and a perfect square)
- $\frac{\text{Total number of factors of } \frac{k}{4}}{2}$ (If k is even and not a perfect square)
- $\frac{(\text{Total number of factors of } \frac{k}{4}) - 1}{2}$ (If it is even and a perfect square)

Tip 10 - Number systems

- Number of digits in $a^b = [b \log_m(a)] + 1$; where m is the base of the number and $[.]$ denotes greatest integer function
- Even number which is not a multiple of 4, can never be expressed as a difference of 2 perfect squares.

Tip 11 - Number systems

- Sum of first n odd numbers is n^2
- Sum of first n even numbers is $n(n+1)$
- The product of the factors of N is given by $N^{a/2}$, where a is the number of factors

Tip 12 - Number systems

- The last two digits of a^2 , $(50 - a)^2$, $(50+a)^2$, $(100 - a)^2$ are same.
- 2^{10n}

When n is odd, the last 2 digits are 24.

When n is even, the last 2 digits are 76.

Tip 13 - Number systems

Divisibility

- Divisibility by 2: Last digit divisible by 2
- Divisibility by 4: Last two digits divisible by 4
- Divisibility by 8: Last three digits divisible by 8
- Divisibility by 16: Last four digit divisible by 16

Tip 14 - Number systems

Divisibility

- Divisibility by 3: Sum of digits divisible by 3
- Divisibility by 9: Sum of digits divisible by 9
- Divisibility by 27: Sum of blocks of 3 (taken right to left) divisible by 27
- Divisibility by 7: Remove the last digit, double it and subtract it from the truncated original number. Check if number is divisible by 7
- Divisibility by 11: (sum of odd digits) - (sum of even digits) should be 0 or divisible by 11

Tip 15 - Number systems

Divisibility properties

- For composite divisors, check if the number is divisible by the factors individually. Hence to check if a number is divisible by 6 it must be divisible by 2 and 3.
- The equation $a^n - b^n$ is always divisible by $a - b$. If n is even it is divisible by $a + b$. If n is odd it is not divisible by $a + b$.
- The equation $a^n + b^n$, is divisible by $a + b$ if n is odd. If n is even it is not divisible by $a + b$.

Tip 16 - Number systems

- Converting from decimal to base b . Let $R_1, R_2 \dots$ be the remainders left after repeatedly dividing the number with b . Hence, the number in base b is given by $\dots R_2 R_1$.
- Converting from base b to decimal - multiply each digit of the number with a power of b starting with the rightmost digit and b^0 .
- A decimal number is divisible by $b-1$ only if the sum of the digits of the number when written in base b are divisible by $b-1$.

Tip 17 - Number systems

Cyclicity

► To find the last digit of a^n find the cyclicity of a . For Ex. if $a=2$, we see that

- $2^1=2$
- $2^2=4$
- $2^3=8$
- $2^4=16$
- $2^5=32$

Hence, the last digit of 2 repeats after every 4th power. Hence cyclicity of 2 = 4. Hence if we have to find the last digit of a^n , The steps are:

1. Find the cyclicity of a , say it is x
2. Find the remainder when n is divided by x , say remainder r
3. Find a^r if $r>0$ and a^x when $r=0$

Tip 18 - Number systems

- $(a + b)(a - b) = (a^2 - b^2)$
- $(a + b)^2 = (a^2 + b^2 + 2ab)$
- $(a - b)^2 = (a^2 + b^2 - 2ab)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Tip 19 - Number systems

- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
- When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Tip 1 – Ratio and Proportion

- Ratio and Proportions is one of the easiest concepts in CAT. It is just an extension of high school mathematics.
- Questions from this concept are mostly asked in conjunction with other concepts like similar triangles, mixtures and alligations.
- Hence fundamentals of this concept are important not just from a stand-alone perspective, but also to answer questions from other concepts

Tip 2 – Ratio and Proportion

- A ratio can be represented as fraction a/b or using the notation $a:b$. In each of these representation 'a' is called the antecedent and 'b' is called the consequent.
- For a ratio to be defined, the quantities of the items should be of same nature. We can not compare the length of the rod to the area of a square.
- However if these quantities are represented in numbers, i.e., length of a rod is a cm and area of a square is b sq.km, we can still define the ratio of these numbers as $a:b$

Tip 3 – Ratio and Proportion

Properties of Ratios :

- A ratio need not be positive. However, if we are dealing with quantities of items, their ratios will be positive. In this concept we will consider only positive ratios.
- A ratio remains the same if both antecedent and consequent are multiplied or divided by the same non-zero number, i.e.,

$$\frac{a}{b} = \frac{pa}{pb} = \frac{qa}{qb}, p, q \neq 0$$

$$\frac{a}{b} = \frac{a/p}{b/p} = \frac{a/q}{b/q}, p, q \neq 0$$

Tip 4 – Ratio and Proportion

- Two ratios in their fraction notation can be compared just as we compare real numbers.

$$\frac{a}{b} = \frac{p}{q} \Leftrightarrow aq = bp$$

$$\frac{a}{b} > \frac{p}{q} \Leftrightarrow aq > bp$$

$$\frac{a}{b} < \frac{p}{q} \Leftrightarrow aq < bp$$

- If antecedent > consequent, the ratio is said to be ratio of greater inequality.
- If antecedent < consequent, the ratio is said to be ratio of lesser inequality.
- If the antecedent = consequent, the ratio is said to be ratio of equality

Tip 5 – Ratio and Proportion

If a , b , x are positive, then

- If $a > b$, then $\frac{a+x}{b+x} < \frac{a}{b}$
- If $a < b$, then $\frac{a+x}{b+x} > \frac{a}{b}$
- If $a > b$, then $\frac{a-x}{b-x} > \frac{a}{b}$
- If $a < b$, then $\frac{a-x}{b-x} < \frac{a}{b}$
- If $\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{d}{s} = \dots$, then $a:b:c:d:\dots = p:q:r:s:\dots$

Tip 6 – Ratio and Proportion

If two ratios a/b and c/d are equal

- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$ (Invertendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$ (Alternendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo-Dividendo)
- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{pa+qb}{ra+sb} = \frac{pc+qd}{rc+sd}$, for all real p, q, r, s such that $pa+qb \neq 0$ and $rc+sd \neq 0$

Tip 7 – Ratio and Proportion

If a, b, c, d, e, f, p, q, r are constants and are not equal to zero

- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$ then each of these ratios is equal to $\frac{a+c+e+..}{b+d+f+..}$
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$ then each of these ratios is equal to _____
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$ then each of these ratios is equal to $\frac{(pna+qnc+rne+..)^{1/n}}{(pnb+qnd+rnf+..)^{1/n}}$
- Duplicate Ratio of a : b is $a^2 : b^2$
- Sub-duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$
- Triplicate Ratio of a : b is $a^3 : b^3$
- Sub-triplicate ratio of a : b is $a^{1/3} : b^{1/3}$

Tip 8 – Ratio and Proportion

Proportions :

A proportion is an equality of ratios. Hence $a:b = c:d$ is a proportion. The first and last terms are called extremes and the other two terms are called means.

If four terms a, b, c, d are said to be proportional, then $a:b = c:d$. If three terms a, b, c are said to be proportional, then $a:b = b:c$

Tip 9 – Ratio and Proportion

Properties of proportions :

If $a:b = c:d$ is a proportion, then

- Product of extremes = product of means i.e., $ad = bc$
- Denominator addition/subtraction: $a:a+b = c:c+d$ and $a:a-b = c:c-d$
- $a, b, c, d, \dots$ are in continued proportion means, $a:b = b:c = c:d = \dots$
- $a:b = b:c$ then b is called mean proportional and $b^2 = ac$
- The third proportional of two numbers, a and b , is c , such that, $a:b = b:c$
- d is fourth proportional to numbers a, b, c if $a:b = c:d$

Tip 10 – Ratio and Proportion

Variations :

- If x varies directly to y , then x is said to be in directly proportional with y and is written as $x \propto y$

$x = ky$ (where k is direct proportionality constant)

$x = ky + C$ (If x depends upon some other fixed constant C)

- If x varies inversely to y , then x is said to be in inversely proportional with y and is written as $x \propto \frac{1}{y}$

$x = k \frac{1}{y}$ (where k is indirect proportionality constant)

$x = k \frac{1}{y} + C$ (If x depends upon some other fixed constant C)

Tip 11 – Ratio and Proportion

Variations :

- If $x \propto y$ and $y \propto z$ then $x \propto z$
- If $x \propto y$ and $x \propto z$ then $x \propto (y \pm z)$
- If $a \propto \frac{1}{b}$ and $b \propto y$ then $ax \propto by$

Tip 1 – Inequalities

- The topic Inequalities is one of the few sections in the quantitative part which can throw up tricky questions. The questions are often asked in conjunction with other sections like ratio and proportion, progressions etc.
- The theory involved in Inequalities is very limited and students should be comfortable with the basics involving addition, multiplication and changing of signs of the inequalities.
- The scope for making an error is high in this section as a minor mistake in calculation (like forgetting the sign) can lead to a completely different answer.

Tip 2 – Inequalities

- The modulus of x , $|x|$ equals the maximum of x and $-x$

$$-|x| \leq x \leq |x|$$

- For any two real numbers 'a' and 'b',

$$|a| + |b| \geq |a + b|$$

$$|a| - |b| \leq |a - b|$$

$$|a \cdot b| = |a| |b|$$

Tip 3 – Inequalities

- For any three real numbers X , Y and Z ; if $X > Y$ then $X+Z > Y+Z$
- If $X > Y$ and
 1. Z is positive, then $XZ > YZ$
 2. Z is negative, then $XZ < YZ$
 3. If X and Y are of the same sign, $1/X < 1/Y$
 4. If X and Y are of different signs, $1/X > 1/Y$

Tip 4 – Inequalities

- For any positive real number, $x + \frac{1}{x} \geq 2$
- For any real number $x > 1$,
$$2 < \left[1 + \frac{1}{x}\right]^x < 2.8.$$

As x increases, the function tends to an irrational number called 'e' which is approximately equal to 2.718

Tip 5 – Inequalities

- If $|x| \leq k$ then the value of x lies between $-k$ and k , or $-k \leq x \leq k$
- If $|x| \geq k$ then $x \geq k$ or $x \leq -k$

Tip 6 – Inequalities

- If $ax^2+bx+c < 0$ then $(x-m)(x-n) < 0$, and if $n > m$, then $m < x < n$
- If $ax^2+bx+c > 0$ then $(x-m)(x-n) > 0$ and if $m < n$, then $x < m$ and $x > n$
- If $ax^2+bx+c > 0$ but $m = n$, then the value of x exists for all values, except x is equal to m ,

i.e., $x < m$ and $x > m$ but $x \neq m$

Tip 1 – Quadratic Equations

- Quadratic Equations is one of the important topics for CAT
- The theory involved in this topic is very simple and students should be comfortable with the some basic formulas and concepts.
- The techniques like option elimination, value assumption can help to solve questions from this topic quickly.
- This pdf covers all the important formulas and concepts related to Quadratic Equations.

Tip 2 – Quadratic Equations

General Quadratic equation will be in the form of $ax^2+bx+c = 0$

The values of 'x' satisfying the equation are called roots of the equation.

- The value of roots, p and q = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Sum of the roots = $p+q = \frac{-b}{a}$
- Product of the roots = $p*q = \frac{c}{a}$
- If c and a are equal then the roots are reciprocal to each other
- If $b = 0$, then the roots are equal and are opposite in sign.

Tip 3 – Quadratic Equations

Let D denote the discriminant, $D = b^2 - 4ac$. Depending on the sign and value of D , nature of the roots would be as follows:

- $D < 0$ and $|D|$ is not a perfect square:

Roots will be in the form of $p+iq$ and $p-iq$ where p and q are the real and imaginary parts of the complex roots. p is rational and q is irrational.

- $D < 0$ and $|D|$ is a perfect square:

Roots will be in the form of $p+iq$ and $p-iq$ where p and q are both rational.

- $D = 0$

Roots are real and equal. $X = -b/2a$

Tip 4 – Quadratic Equations

- $D > 0$ and D is not a perfect square:
Roots are conjugate surds
- $D > 0$ and D is a perfect square:
Roots are real, rational and unequal

Tip 5 – Quadratic Equations

Signs of the roots: Let P be product of roots and S be their sum

- $P > 0, S > 0$: Both roots are positive
- $P > 0, S < 0$: Both roots are negative
- $P < 0, S > 0$: Numerical smaller root is negative and the other root is positive
- $P < 0, S < 0$: Numerical larger root is negative and the other root is positive

Tip 6 – Quadratic Equations

- Minimum and maximum values of $ax^2+bx+c = 0$
- If $a > 0$: minimum value = $\frac{4ac-b^2}{4a}$ and occurs at $x = -b/2a$
- If $a < 0$: maximum value = $\frac{4ac-b^2}{4a}$ and occurs at $x = -b/2a$

Tip 7 – Quadratic Equations

If $A_n X^n + A_{n-1} X^{n-1} + \dots + A_1 X + A_0 = 0$, then

- Sum of the roots = $-A_{n-1}/A_n$
- Sum of roots taken two at a time = A_{n-2}/A_n
- Sum of roots taken three at a time = $-A_{n-3}/A_n$ and so on
- Product of the roots = $[(-1)^n A_0] / A_n$

Tip 8 – Quadratic Equations

Finding a quadratic equation:

- If roots are given: $(x-a)(x-b)=0 \Rightarrow x^2-(a+b)x+ab = 0$

If sum s and product p of roots are given: $x^2-sx+p = 0$

- If roots are reciprocals of roots of equation $ax^2+bx+c = 0$, then equation is $cx^2+bx+a = 0$
- If roots are k more than roots of $ax^2+bx+c = 0$ then equation is $a(x-k)^2+b(x-k)+c = 0$
- If roots are k times roots of $ax^2+bx+c = 0$ then equation is $a(x/k)^2+b(x/k)+c = 0$

Tip 9 – Quadratic Equations

- Descartes Rules: A polynomial equation with n sign changes can have a maximum of n positive roots. To find the maximum possible number of negative roots, find the number of positive roots of $f(-x)$.
- An equation where highest power is odd must have at least one real root

Tip 1 – Linear equations

- Linear equations is one of the foundation topics in the Quant section of CAT.
- Hence, fundamentals of this concept are useful in solving the questions of the other topics by assuming the unknown values as variables.
- Be careful of silly mistakes in this topic as that is how students generally lose marks here.
- Generally, the number of equations needed to solve the given problem is equal to the number of variables

Tip 2 – Linear equations

- A linear equation is an equation which gives straight line when plotted on a graph.
- Linear equations can be of one variable or two variable or three variable.
- Let a , b , c and d are constants and x , y and z are variables. A general form of single variable linear equation is $ax+b = 0$.
- A general form of two variable linear equation is $ax+by = c$.
- A general form of three variable linear equation is $ax+by+cz = d$.

Tip 3 – Linear equations

Equations with two variables:

- Consider two equations $ax+by = c$ and $mx+ny = p$. Each of these equations represent two lines on the x-y co-ordinate plane. The solution of these equations is the point of intersection.
- If $\frac{a}{m} = \frac{b}{n} \neq \frac{c}{p}$ then the slope of the two equations is equal and so they are parallel to each other. Hence, no point of intersection occurs. Therefore no solution.
- If $\frac{a}{m} \neq \frac{b}{n}$ then the slope is different and so they intersect each other at a single point. Hence, it has a single solution.
- If $\frac{a}{m} = \frac{b}{n} = \frac{c}{p}$ then the two lines are same and they have infinite points common to each other. So, infinite solutions occurs

Tip 4 – Linear equations

General Procedure to solve linear equations:

- Aggregate the constant terms and variable terms
- For equations with more than one variable, eliminate variables by substituting equations in their place.
- Hence, for two equations with two variables x and y , express y in terms of x and substitute this in the other equation.
- For Example, let $x+y = 14$ and $x+4y = 26$ then $x = 14-y$ (from equation 1) substituting this in equation 2, we get $14-y+4y = 26$. Hence, $y = 4$ and $x = 10$.

Tip 5 – Linear equations

General Procedure to solve linear equations:

For equations of the form $ax+by = c$ and $mx+ny = p$, find the LCM of b and n . Multiply each equation with a constant to make the y term coefficient equal to the LCM. Then subtract equation 2 from equation 1.

Example:

Let $2x+3y = 13$ and $3x+4y = 18$ are the given equations (1) and (2).

- LCM of 3 and 4 is 12.
- Multiplying (1) by 4 and (2) by 3, we get $8x+12y = 52$ and $9x+12y = 54$.
- $(2)-(1)$ gives $x=2$, $y=3$

Tip 6 – Linear equations

- If the system of equations has n variables with $n-1$ equations then the solution is indeterminate
- If system of equations has n variables with $n-1$ equations with some additional conditions like the variables are integers then the solution may be determinate
- If system of equations has n variables with $n-1$ equations then some combination of variables may be determinable.
- For example, if $ax+by+cz = d$ and $mx+ny+pz = q$, if a, b, c are in Arithmetic progression and m, n and p are in AP then the sum $x+y+z$ is determinable.

Tip 7 – Linear equations

Equations with three variables:

Let the equations be $a_1x+b_1y+c_1z = d_1$, $a_2x+b_2y+c_2z = d_2$ and $a_3x+b_3y+c_3z = d_3$. Here we define the following matrices.

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad D_x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix} \quad D_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix} \quad D_z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

- If Determinant of $D \neq 0$, then the equations have a unique solution.
- If Determinant of $D = 0$, and at least one but not all of the determinants D_x , D_y or D_z is zero, then no solution exists.
- If Determinant of $D = 0$, and all the three of the determinants D_x , D_y and D_z are zero, then there are infinitely many solution exists.
- Determinant can be calculated by $D = a_1(b_2c_3-c_2b_3)-b_1(a_2c_3-c_2a_3)+c_1(a_2b_3-b_2a_3)$

Previous year Quant's for CAT Preparation



Dear CAT Aspirants,

It's a tough journey to reach where we want to be in the career. It takes lots of efforts, struggles, long studying hours to crack CAT – One of the toughest management entrance exams in the country. Being one amongst the premier B School, Jaipuria Institute of Management accepts CAT score as one of its component in the selection process.

The process being very stringent, a limited number of aspirants are able to make through the selection process. In our endeavor to encourage and nurture management aspirants, we share lots of CAT related material in form of articles, ebooks, videos and even organize free career-oriented workshops. So, this time here we are with Reading Comprehension with their solutions.

Happy Reading!



Quant

Question 1

If Y is the number such that $2^{Y^2 \log_3 5} = 5 \log_2 3$, then Y equals to:

- A $\log_2\left(\frac{1}{5}\right)$
- B $\log_2\left(\frac{1}{3}\right)$
- C $-\log_2\left(\frac{1}{5}\right)$
- D $-\log_2\left(\frac{1}{3}\right)$

Answer: B

Question 2

A gentleman decided to treat a few children in the following manner. He gives half of his total stock of toffees and one extra to the first child, and then the half of the remaining stock along with one extra to the second and continues giving away in this fashion. His total stock exhausts after he takes care of 5 children. How many toffees were there in his stock initially?

Answer:62

Question 3

How many 3-digit numbers are there, for which the product of their digits is more than 2 but less than 7?

Answer:21

Question 4

is:

The number of real-valued solutions of the equation $2^x + 2^{-x} = 2 - (x - 2)^2$

- A 1
- B 2
- C infinite
- D 0

Answer: D

Question 5

How many distinct positive integer-valued solutions exist to the equation $(X^2 - 7x + 11)^{(X^2 - 13x + 42)} = 1$?

- A 8
- B 4
- C 2
- D 6

Answer: D

Question 6

A solid right circular cone of height 27 cm is cut into two pieces along a plane parallel to its base at a height of 18 cm from the base. If the difference in volume of the two pieces is 225 cc, the volume, in cc, of the original cone is

- A 243
- B 232
- C 256
- D 264

Answer: A

Question 7

The area of the region satisfying the inequalities $|x| - y \leq 1, y \geq 0$ and $y \leq 1$ is

Answer: 3

Question 8

On a rectangular metal sheet of area 135 sq in, a circle is painted such that the circle touches two opposite sides. If the area of the sheet left unpainted is two-thirds of the painted area then the perimeter of the rectangle in inches is

- A $3\sqrt{\pi}\left(5 + \frac{12}{\pi}\right)$
- B $4\sqrt{\pi}\left(3 + \frac{9}{\pi}\right)$
- C $3\sqrt{\pi}\left(\frac{5}{2} + \frac{6}{\pi}\right)$
- D $5\sqrt{\pi}\left(3 + \frac{9}{\pi}\right)$

Answer: A

Question 9

A circle is inscribed in a rhombus with diagonals 12 cm and 16 cm. The ratio of the area of circle to the area of rhombus is

- A $\frac{6\pi}{25}$
- B $\frac{5\pi}{18}$
- C $\frac{3\pi}{25}$
- D $\frac{2\pi}{15}$

Answer: A

Question 10

Among 100 students, x_1 have birthdays in january, x_2 have girthdays in february, and so on. If $x_0 = \max(x_1, x_2, \dots, x_{12})$, then the smallest possible value of x_0 is

- A 8
- B 9

C 10

D 12

Answer: B

Question 11

A straight road connects points A and B. Car 1 travels from A to B and Car 2 travels from B to A, both leaving at the same time. After meeting each other, they take 45 minutes and 20 minutes, respectively, to complete their journeys. If Car 1 travels at the speed of 60 km/hr, then the speed of Car 2, in km/hr, is

A 100

B 90

C 80

D 70

Answer: B

Question 12

A person spent Rs 50000 to purchase a desktop computer and a laptop computer. He sold the desktop at 20% profit and the laptop at 10% loss. If overall he made a 2% profit then the purchase price, in rupees, of the desktop is

Answer: 20000

Question 13

A solution, of volume 40 litres, has dye and water in the proportion 2 : 3. Water is added to the solution to change this proportion to 2 : 5. If one fourth of this diluted solution is taken out, how many litres of dye must be added to the remaining solution to bring the proportion back to 2 : 3?

Answer: 8

Question 14

If $x = (4096)^{7+4\sqrt{3}}$, then which of the following equals to 64?

A $\frac{x^7}{x^{2\sqrt{3}}}$

B $\frac{x^{\frac{7}{2}}}{x^{\frac{4}{\sqrt{3}}}}$

C $\frac{x^{\frac{7}{2}}}{x^{2\sqrt{3}}}$

D $\frac{x^7}{x^{4\sqrt{3}}}$

Answer: C

Question 15

An alloy is prepared by mixing three metals A, B and C in the proportion 3 : 4 : 7 by volume. Weights of the same volume of the metals A, B and C are in the ratio 5 : 2 : 6. In 130 kg of the alloy, the weight, in kg, of the metal C is

- A 48
- B 84
- C 70
- D 96

Answer: B

Question 16

The number of distinct real roots of the equation $(x + \frac{1}{x})^2 - 3(x + \frac{1}{x}) + 2 = 0$ equals

Answer:1

Question 17

If $\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$, then y equals

Answer:36

Question 18

Leaving home at the same time, Amal reaches office at 10:15 am if he travels at 8 km/hr, and at 9:40 am if he travels at 15 km/hr. Leaving home at 9.10 am, at what speed, in km/hr, must he travel so as to reach office exactly at 10 am?

- A 13
- B 12
- C 14
- D 11

Answer: B

Question 19

A train travelled at one-thirds of its usual speed, and hence reached the destination 30 minutes after the scheduled time. On its return journey, the train initially travelled at its usual speed for 5 minutes but then stopped for 4 minutes for an emergency. The percentage by which the train must now increase its usual speed so as to reach the destination at the scheduled time, is nearest to

- A 50
- B 58
- C 67
- D 61

Answer: C

Question 20

The mean of all 4-digit even natural numbers of the form 'aabb', where $a > 0$, is

- A 4466
- B 5050
- C 4864
- D 5544

Answer: D

Question 21

Two persons are walking beside a railway track at respective speeds of 2 and 4 km per hour in the same direction. A train came from behind them and crossed them in 90 and 100 seconds, respectively. The time, in seconds, taken by the train to cross an electric post is nearest to

- A 87
- B 82
- C 78
- D 72

Answer: B

Question 22

If a , b and c are positive integers such that $ab = 432$, $bc = 96$ and $c < 9$, then the smallest possible value of $a + b + c$ is

- A 49
- B 56
- C 59
- D 46

Answer: D

Question 23

In a group of people, 28% of the members are young while the rest are old. If 65% of the members are literates, and 25% of the literates are young, then the percentage of old people among the illiterates is nearest to

- A 62
- B 55
- C 59
- D 66

Answer: D

Question 24

Veeru invested Rs 10000 at 5% simple annual interest, and exactly after two years, Joy invested Rs 8000 at 10% simple annual interest. How many years after Veeru's investment, will their balances, i.e., principal plus accumulated interest, be equal?

Answer:12

Question 25

If $f(5 + x) = f(5 - x)$ for every real x , and $f(x) = 0$ has four distinct real roots, then the sum of these roots is

A 0

B 40

C 10

D 20

Answer: D

Question 26

Let A, B and C be three positive integers such that the sum of A and the mean of B and C is 5. In addition, the sum of B and the mean of A and C is 7. Then the sum of A and B is

A 5

B 4

C 6

D 7

Answer: C

Instructions

For the following questions answer them individually

Question 1

Two alcohol solutions, A and B, are mixed in the proportion 1:3 by volume. The volume of the mixture is then doubled by adding solution A such that the resulting mixture has 72% alcohol. If solution A has 60% alcohol, then the percentage of alcohol in solution B is

A 90%

B 94%

C 92%

D 89%

Answer: C

Question 2

A batsman played $n + 2$ innings and got out on all occasions. His average score in these $n + 2$ innings was 29 runs and he scored 38 and 15 runs in the last two innings. The batsman scored less than 38 runs in each of the first n innings. In these n innings, his average score was 30 runs and lowest score was x runs. The smallest possible value of x is

A 4

B 3

C 2

D 1

Answer: C

Question 3

Let m and n be positive integers, If $x^2 + mx + 2n = 0$ and $x^2 + 2nx + m = 0$ have real roots, then the smallest possible value of $m + n$ is

A 7

B 6

C 8

D 5

Answer: B

Question 4

A contractor agreed to construct a 6 km road in 200 days. He employed 140 persons for the work. After 60 days, he realized that only 1.5 km road has been completed. How many additional people would he need to employ in order to finish the work exactly on time?

Answer:40

Question 5

If $x_1 = -1$ and $x_m = x_{m+1} + (m + 1)$ for every positive integer m, then X_{100} equals

A -5050

B -5151

C -5051

D -5150

Answer: A

Question 6

If $\log_a 30 = A$, $\log_a \left(\frac{5}{3}\right) = -B$ and $\log_2 a = \frac{1}{3}$, then $\log_3 a$ equals

A $\frac{2}{A+B-3}$

B $\frac{2}{A+B} - 3$

C $\frac{A+B}{2} - 3$

D $\frac{A+B-3}{2}$

Answer: A

Question 7

Dick is thrice as old as Tom and Harry is twice as old as Dick. If Dick's age is 1 year less than the average age of all three, then Harry's age, in years, is

Answer:18

Question 8

Vimla starts for office every day at 9 am and reaches exactly on time if she drives at her usual speed of 40 km/hr. She is late by 6 minutes if she drives at 35 km/hr. One day, she covers two-thirds of her distance to office in one-thirds of her usual time to reach office, and then stops for 8 minutes. The speed, in km/hr, at which she should drive the remaining distance to reach office exactly on time is

A 29

B 26

C 28

D 27

Answer: C

Question 9

Let m and n be natural numbers such that n is even and $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$. Then $m - 2n$ equals

A 3

B 1

C 2

D 4

Answer: B

Question 10

How many integers in the set $\{100, 101, 102, \dots, 999\}$ have at least one digit repeated?

Answer: 252

Question 11

In the final examination, Bishnu scored 52% and Asha scored 64%. The marks obtained by Bishnu is 23 less, and that by Asha is 34 more than the marks obtained by Ramesh. The marks obtained by Geeta, who scored 84%, is

A 357

B 417

C 439

D 399

Answer: D

Question 12

If a, b, c are non-zero and $14^a = 36^b = 84^c$, then $6b\left(\frac{1}{c} - \frac{1}{a}\right)$ is equal to

Answer: 3

Question 13

A person invested a certain amount of money at 10% annual interest, compounded half-yearly. After one and a half years, the interest and principal together became Rs.18522. The amount, in rupees, that the person had invested is

Answer: 16000

Question 14

A man buys 35 kg of sugar and sets a marked price in order to make a 20% profit. He sells 5 kg at this price, and 15 kg at a 10% discount. Accidentally, 3 kg of sugar is wasted. He sells the remaining sugar by raising the marked price by p percent so as to make an overall profit of 15%. Then p is nearest to

- A 22
- B 35
- C 25
- D 31

Answer: C

Question 15

The points (2,1) and (-3,-4) are opposite vertices of a parallelogram. If the other two vertices lie on the line $x + 9y + c = 0$, then c is

- A 12
- B 13
- C 15
- D 14

Answer: D

Question 16

A and B are two railway stations 90 km apart. A train leaves A at 9:00 am, heading towards B at a speed of 40 km/hr. Another train leaves B at 10:30 am, heading towards A at a speed of 20 km/hr. The trains meet each other at

- A 11 : 45 am
- B 11 : 20 am
- C 11 : 00 am
- D 10 : 45 am

Answer: C

Question 17

Let N , x and y be positive integers such that $N = x + y$, $2 < x < 10$ and $14y < y < 23$. If $N > 25$, then how many distinct values are possible for N ?

Answer: 6

Question 18

Let k be a constant. The equations $kx + y = 3$ and $4x + ky = 4$ have a unique solution if and only if

A $|k| \neq 2$

B $|k| = 2$

C $k \neq 2$

D $k = 2$

Answer: A

Question 19

How many of the integers 1, 2, ..., 120, are divisible by none of 2, 5 and 7?

A 42

B 41

C 40

D 43

Answer: B

Question 20

How many pairs (a, b) of positive integers are there such that $a \leq b$ and $ab = 4^{2017}$?

A 2018

B 2019

C 2017

D 2020

Answer: A

Question 21

Anil, Sunil, and Ravi run along a circular path of length 3 km, starting from the same point at the same time, and going in the clockwise direction. If they run at speeds of 15 km/hr, 10 km/hr, and 8 km/hr, respectively, how much distance in km will Ravi have run when Anil and Sunil meet again for the first time at the starting point?

A 4.8

B 4.6

C 5.2

D 4.2

Answer: A

Question 22

In a trapezium $ABCD$, AB is parallel to DC , BC is perpendicular to DC and $\angle BAD = 45^\circ$. If $DC = 5\text{ cm}$, $BC = 4\text{ cm}$, the area of the trapezium in sq cm is

Answer: 28

Question 23

The area, in sq. units, enclosed by the lines $x = 2$, $y = |x - 2| + 4$, the X-axis and the Y-axis is equal to

A 10

B 6

C 8

D 12

Answer: A

Question 24

If $(x + y) = f(x)f(y)$ and $f(5) = 4$, then $f(10) - f(-10)$ is equal to

A 14.0625

B 0

C 15.9375

D 3

Answer: C

Question 25

$\frac{2 \times 4 \times 8 \times 16}{(\log_2 4)^2 (\log_4 8)^3 (\log_8 16)^4}$ equals

Answer: 24

Question 26

The vertices of a triangle are (0,0), (4,0) and (3,9). The area of the circle passing through these three points is

A $\frac{14\pi}{3}$

B $\frac{123\pi}{7}$

C $\frac{12\pi}{5}$

D $\frac{205\pi}{9}$

Answer: D

Previous year Quant's for CAT

CAT 2019

Question 1:

In a class, 60% of the students are girls and the rest are boys. There are 30 more girls than boys. If 68% of the students, including 30 boys, pass an examination, the percentage of the girls who do not pass is [TITA]

Correct Answer. 20

Given that there are 60 % girls and 40 % boys

It is also given that there are 30 more girls than boys.

So, $(60\% - 40\%)$ of total class strength = 30 students

$\Rightarrow 20\%$ of total class strength = 30 students

$\Rightarrow$ Total class strength = $30 \times 5 = 150$ students

It is also given that 68% of students pass the the examination, which includes 30 boys

So, Number of students passed = 68% of Total students

$\Rightarrow$ Number of students passed = $68/100 \times 150 = 102$ students

$\Rightarrow$ Since the number of boys passed is 30,

$\Rightarrow$ Number of girls passed = $102 - 30 = 72$

Total number of girls = $60/100 \times 150 = 90$

Total number of girls = Girls who passed + Girls who did not pass

Girls who did not pass = $90 - 72 = 18$

Percentage of girls who did not pass = $18/90 \times 100 = 20\%$

Question 2:

With rectangular axes of coordinates, the number of paths from (1,1) to (8,10) via (4,6), where each step from any point (x,y) is either to (x,y+1) or to (x+1,y) is [TITA]

Correct Answer. 3920

Let us first consider travelling from (1,1) to (4, 6)

This means, Travelling from 1 to 4 units in the x axis $\rightarrow$ 3 horizontal (h h h)

Previous year Quant's for CAT

CAT 2019

And travelling from 1 to 6 units in the y axis $\rightarrow$ 5 vertical movements
(v v v v v)

No matter how we proceed, reaching from (1,1) to (4,6) requires 5 vertical movements and 3 horizontal movements.

\So, Number of paths to travel from (1,1) to (4,6) = Number of ways of arranging (h h h v v v v v)

Number of ways of arranging (h h h v v v v v) = $8!/3!5!$

imilarly, travelling from (4, 6) to (8, 10) requires 4 horizontal movements and 4 vertical movements

Number of ways of arranging (h h h h v v v v) = $8!/4!4!$

Total number of paths = $8!/3!5! \times 8!/4!4! = 3920$

Question 3:

A club has 256 members of whom 144 can play football, 123 can play tennis, and 132 can play cricket. Moreover, 58 members can play both football and tennis, 25 can play both cricket and tennis, while 63 can play both football and cricket. If every member can play at least one game, then the number of members who can play only tennis is

- A. 32
- B. 43
- C. 38
- D. 45

Correct Answer. 43

From observing the data given, we find that it is a closed 3 set Venn diagram. Let the three sports be F, T and C for Football, Tennis and Cricket respectively $n(F \cup T \cup C) = 256$, $n(F) = 144$, $n(T) = 123$, $n(C) = 132$, $n(F \cap T) = 58$, $n(C \cap T) = 25$, $n(F \cap C) = 63$

Previous year Quant's for CAT

CAT 2019

We know, the product of rectangles formed by two intersecting chords are always equal

$$\text{So, } AE \times EB = CE \times ED$$

$$AE \times EB = 7 \times 15$$

$$\text{Also, we know that } AB = 20.5 \text{ ems} = AE + EB$$

So, the Sum of AE and EB must be 20.5 and their product must be equal to $7 \times 15 = 105$

The numbers must be close to each other, for their sum to be 20.5

From trial and error, we find their values to be 10 and 10.5 ems respectively So, $AE \times EB = 7 \times 15$

$$10 \times 10.5 = 7 \times 15$$

$$\text{Difference} = 10.5 - 10 \text{ ems} = 0.5 \text{ ems}$$

Question 5:

Meena scores 40% in an examination and after review, even though her score is increased by 50%, she fails by 35 marks. If her post-review score is increased by 20%, she will have 7 marks more than the passing score. The percentage score needed for passing the examination is

A. 75

8.86

C.60

D.70

Correct Answer. 70

Meena scores 40 % in an exam

After review, she scores 50 % more => Increase of 50 % from 40 % = $40\% + 20\% = 60\%$

She fails by 35 marks, by scoring 60%

$$60\% \text{ score} = \text{Pass mark} - 35 \text{ ----- (1)}$$

If her post review score is increased by 20%, she would have 7 more than the pass mark.

Previous year Quant's for CAT

CAT 2019

We know, the product of rectangles formed by two intersecting chords are always equal

$$\text{So, } AE \times EB = CE \times ED$$

$$AE \times EB = 7 \times 15$$

Also, we know that $AB = 20.5 \text{ ems} = AE + EB$

So, the Sum of AE and EB must be 20.5 and their product must be equal to $7 \times 15 = 105$

The numbers must be close to each other, for their sum to be 20.5

From trial and error, we find their values to be 10 and 10.5 ems respectively So, $AE \times EB = 7 \times 15$

$$10 \times 10.5 = 7 \times 15$$

$$\text{Difference} = 10.5 - 10 \text{ ems} = 0.5 \text{ ems}$$

Question 5:

Meena scores 40% in an examination and after review, even though her score is increased by 50%, she fails by 35 marks. If her post-review score is increased by 20%, she will have 7 marks more than the passing score. The percentage score needed for passing the examination is

A. 75

B. 86

C. 60

D. 70

Correct Answer. 70

Meena scores 40 % in an exam

After review, she scores 50 % more => Increase of 50 % from 40 % = $40\% + 20\% = 60\%$

She fails by 35 marks, by scoring 60%

$$60\% \text{ score} = \text{Pass mark} - 35 \text{ ----- (1)}$$

If her post review score is increased by 20%, she would have 7 more than the pass mark.

Previous year Quant's for CAT

CAT 2019

20% of 60% = 12 %

So, 60% + 12% = 72% of marks = Pass mark + 7 ----- (2)

So, 12% marks = 35 + 7 (5: 1 ratio)

So, similarly 12% can be re written as 10 % and 2 %
maintaining the 5:1 ratio)

Hence the pass percentage = 60 % + 10 % = 70%

Question 6:

Corners are cut off from an equilateral triangle T to produce a regular hexagon H. Then, the ratio of the area of H to the area of T is

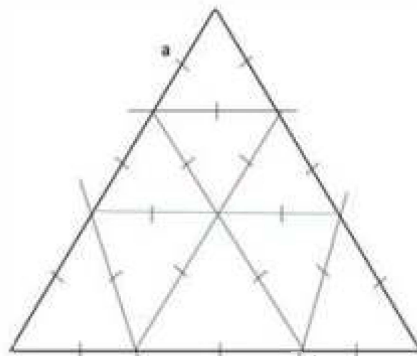
A. 5 : 6

B. 3 : 4

C. 2 : 3

D. 4 : 5

Correct Answer. 2.3



Construct an Equilateral triangle and cut off equal lengths from all three sides. So we obtain a hexagon and three equilateral triangles of side 'a' in return.

We know that a hexagon comprises of 6 Equilateral Triangles. Since the side of the hexagon is also 'a', we obtain 6 equilateral triangles in return. Thus, we have a total of $6 + 3 = 9$ equilateral triangles of side 'a'

Ratio of area of Hexagon with Area of Triangle = $6 : 9 = 2 : 3$

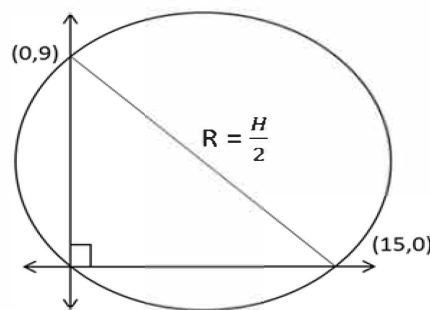
Previous year Quant's for CAT

CAT 2019

Question 7:

Let T be the triangle formed by the straight line $3x + 5y - 45 = 0$ and the coordinate axes. Let the circumcircle of T have radius of length L , measured in the same unit as the coordinate axes. Then, the integer closest to L is [TITA]

Correct Answer. 9



We know that the equation of the straight line is $3x + 5y = 45$ $x/15 + y/9 = 1$

The intercepts are $(15,0)$ and $(0,9)$ respectively

Since it's a right-angled triangle, we know that Circumradius (R)
=hypotenuse/2

$$\text{Circumradius} = \frac{\sqrt{9^2 + 5^2}}{2} = \frac{3\sqrt{34}}{2} \approx 9.1$$

We know that $\sqrt{34}$ is approximately equal to 6

So, from trial and error to find the closest number, we find that the value of Circumradius is very close to 9

So, the integer closest to $L = 9$

Question 8:

For any positive integer n , let $f(n) = n(n + 1)$ if n is even, and $f(n) = n + 3$ if n is odd. If m is a positive integer such that $8f(m + 1) - f(m) = 2$, then m equals [TITA]

Previous year Quant's for CAT

CAT 2019

Correct Answer. 10

If n is even, $f(n) = n(n + 1)$

So, $f(2) = 2(2+1) = 2(3) = 6$

If n is odd, $f(n) = n + 3$

$f(1) = 1 + 3 = 4$

It is given that, $8 \times f(m+1) - f(m) = 2$

So, m can either be even or odd

Case-1: If m were even and $m+1$ odd

So, $8 \times f(m+1) - f(m) = 2$

$8(m + 4) - m(m + 1) = 2$

$8m + 32 - m^2 - m = 2$

$m^2 - 7m - 30 = 0$

$(m-10)(m+3) = 0$

$m = 10$ or -3

$m = 10$, since m is positive

Case-2: If m were even and $m+1$ odd

$8 \times f(m+1) - f(m) = 2$

$8(m + 1)(m + 2) - (m + 3) = 2$

Now, let us substitute $m = 1$ which is the minimum possible value

$8(1 + 1)(1 + 2) - (1 + 3) = 2$

Case 2 does not work

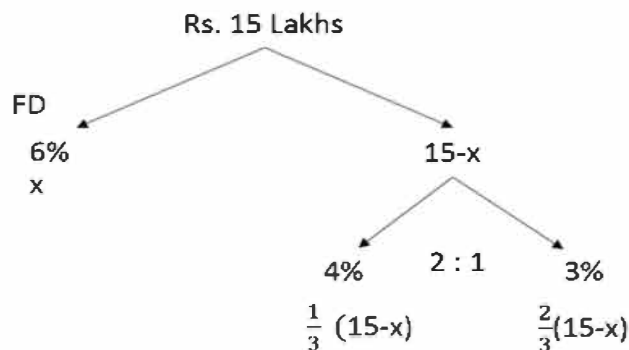
Question 9:

A person invested a total amount of Rs 15 lakh. A part of it was invested in a fixed deposit earning 6% annual interest, and the remaining amount was invested in two other deposits in the ratio 2 : 1, earning annual interest at the rates of 4% and 3%, respectively. If the total annual interest income is Rs 76000 then the amount (in Rs lakh) invested in the fixed deposit was [TITA]

Previous year Quant's for CAT

CAT 2019

Correct Answer: 9



Let Rs x be invested in FD.

So, Remaining amount = Rs. 15 Lakhs - x , which is invested in the ratio 2 : 1 So, each investment would have $\frac{1}{3}\{15-x\}$ and $\frac{2}{3}\{15-x\}$ investments respect So, Total annual interest income = 6% of x + 4% of $\frac{1}{3}(15-x)$ + 3% of $\frac{2}{3}(15-x)$ = 76000

Rewriting Rs.76000 as Rs.076 Lakhs,

$$6\% \text{ of } x + 4\% \text{ of } \frac{1}{3}(15-x) + 3\% \text{ of } \frac{2}{3}(15-x) = 0.76$$

Multiply both sides by 100,

$$6x + \frac{4}{3}\{15-x\} + 2(15-x) = 76$$

$$6x + \frac{11}{3}\{15-x\} = 76$$

$$165 + 11x - 11x = 228$$

$$7x = 63$$

$$x = 9 \text{ (or) Rs.9 Lakhs}$$

Question 10:

The product of two positive numbers is 616. If the ratio of the difference of their cubes to the cube of their difference is 157 : 3, then the sum of the two numbers is

- A. 50
- B. 85
- C. 95
- D. 58

Previous year Quant's for CAT

CAT 2019

Correct Answer. 50

Let the two numbers be a and b

From the question, we know

$$a \times b = 616$$

$$\text{and } \frac{a^2 - b^2}{(a-b)^2} = \frac{157}{3}$$

$$\frac{(a-b)(a^2 + ab + b^2)}{(a-b)^2} = \frac{157}{3}$$

$$\frac{(a^2 + ab + b^2)}{a^2 - 2ab + b^2} = \frac{157}{3}$$

$$\frac{(a^2 - 2ab + b^2) + 3ab}{(a^2 - 2ab + b^2)} = \frac{157}{3}$$

$$\frac{3ab}{(a-b)^2} = \frac{157}{3} - 1 = \frac{154}{3}$$

$$\frac{3ab}{(a-b)^2} = \frac{154}{3}$$

$$\frac{3(616)}{(a-b)^2} = \frac{154}{3}$$

$$(a-b)^2 = 3 \times 3 \times 4 = 36$$

$$a - b = 6, ab = 616$$

616 can be rewritten as 22×28 , where $28 - 22 = 6$ using trial and error So, $a + b = 28 + 22 = 50$

Question 11 :

On selling a pen at 5% loss and a book at 15% gain, Karim gains Rs. 7. If he sells the pen at 5% gain and the book at 10% gain, he gains Rs. 13. What is the cost price of the book in Rupees?

Previous year Quant's for CAT

CAT 2019

- A.80
- 8.85
- C. 100
- D.95

Correct Answer. 80

From the question, let us frame the following equations

$$-P \times 5\% + b \times 15\% = 7 \text{ --- (1)}$$

$$P \times 5\% + b \times 10\% = 13 \text{ --- (2)}$$

(+)

$$b \times (25\%) = 20$$

$$\text{Price of Book} = b = \text{Rs. } 80$$

Question 12 :

If $a_1 + a_2 + a_3 + \dots + a_n = 3\{2^{n+1} - 2\}$, then a_{11} equals [TITA]

Correct Answer. 6144

Let $n = 1$,

$$a_1 = 3(2^{1+1} - 2) = 3(4 - 2) = 6 = 3 \times 2^1$$

$$a_2 = 3(2^{2+1} - 2) = 3(8 - 2) = 18, a_2 = 18 - 6 = 12 = 3 \times 2^2$$

$$a_3 = 3(2^{3+1} - 2) = 3(16 - 2) = 42, a_3 = 42 - 12 - 6 = 24 = 3 \times 2^3$$

$$\text{So, } a_{11} = 3 \times 2^{11} = 3 \times 2048$$

$$a_{11} = 6144$$

Question 13 :

The number of the real roots of the equation $2\cos(x(x+1)) = 2x + 2-x$ is

Previous year Quant's for CAT

CAT 2019

- A. 0
- B. infinite
- C. 1
- D. 2

Correct Answer. 1

$$2\cos(x(x+1)) = 2x + 2^{-x}$$

Consider RHS, $2x + 2^{-x}$

This is of the form, $y + 1/y$

We know that $y + 1/y \geq 2$ when y is positive and $y + 1/y \leq -2$ when y is negative

Here, since we are dealing with $2x$, $y + 1/y \geq 2$ is considered respectively So,

$$2\cos(x(x+1)) \geq 2$$

$$\text{Substitute } x = 0, 2x + 2^{-x} = 20 + 20 = 2$$

$$2\cos(x(x+1)) = 2\cos(0(0+1)) = 2\cos(0) = 2 \quad x = 0 \text{ is a valid solution}$$

Any value other than this won't work.

Therefore, only $x = 0$ works and there is only one real solution

Question 14 :

The income of Amala is 20% more than that of Bimala and 20% less than that of Kamala. If Kamala's income goes down by 4% and Bimala's goes up by 10%, then the percentage by which Kamala's income would exceed Bimala's is nearest to

- A. 28
- B. 29
- C. 31
- D. 32

Correct Answer. 31

Let Bimala's income be B . Amala's income is 20% more than B . So,

$$A = 1.2B$$

Similarly, Amala's income is 20% less than that of Kamala.

$$A = 0.8K$$

Previous year Quant's for CAT

CAT 2019

Therefore, $1.28 = 0.8K$ (or) $K = 3/28$

Now, Simala's new income goes up by 10% = 1.18

Kamala's income goes down by 4%

4% (1.58) = 0.068

Therefore, Kamala's new income is = $1.58 - 0.068 = 1.448$

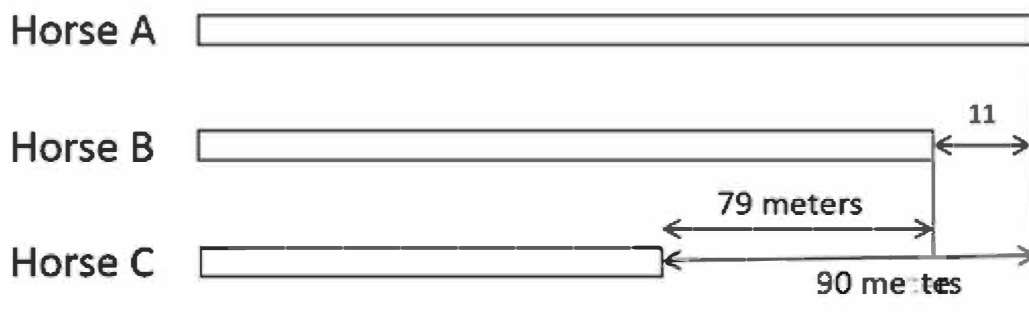
So, $K = 1.448$ and $S = 1.18$

Percentage change = $0.34 / 1.1 \times 100 = 31\%$

Question 15:

In a race of three horses, the first beat the second by 11 metres and the third by 90 metres. If the second beat the third by 80 metres, what was the length, in metres, of the racecourse? [TITA]

Correct Answer. 880



A beats B by 11 meters. When B completes the 11 meters, there is a lead of 80 meters to C

So, C must have travelled only $90 - 80 = 10$ meters

When B travels 11 meters, C travels only 10 meters

Ratio of distance travelled by second and third horse are $1x$ and $10x$ respectively

We know that the second horse beats the third horse by 80 meters. So,
Length of the track = Distance travelled by the second horse

$= 11 \times 80 = 880$ meters

Previous year Quant's for CAT

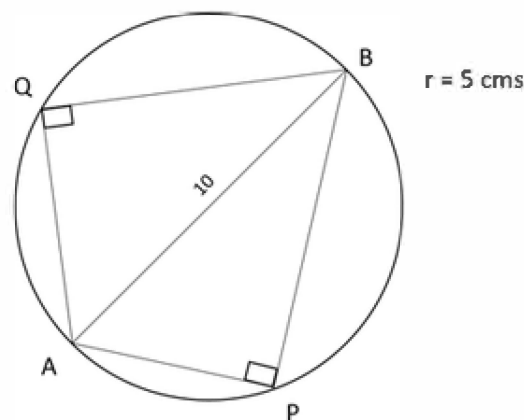
CAT 2019

Question 16:

AB is a diameter of a circle of radius 5 cm. Let P and Q be two points on the circle so that the length of PB is 6 cm, and the length of AP is twice that of AQ. Then the length, in cm, of QB is nearest to

- A. 8.5
- B. 9.3
- C. 9.1
- D. 7.8

Correct Answer: 9.1



We know that angle APB = AQB = 90°

We know PB = 6, AB = 10 and AP = 2AQ

From using Pythagorean triplet, we find that AP = 8 cm

So, AQ = 4 cm

$$AQ^2 + QB^2 = AB^2$$

$$16 + QB^2 = 100$$

$$QB^2 = 84$$

$$QB = 9.1$$

Question 17:

One can use three different transports which move at 10, 20, and 30 kmph, respectively. To reach from A to B, Amal took each mode of transport $\frac{1}{3}$ of his total journey time, while Bimal took each mode of transport $\frac{1}{3}$ of the total distance.

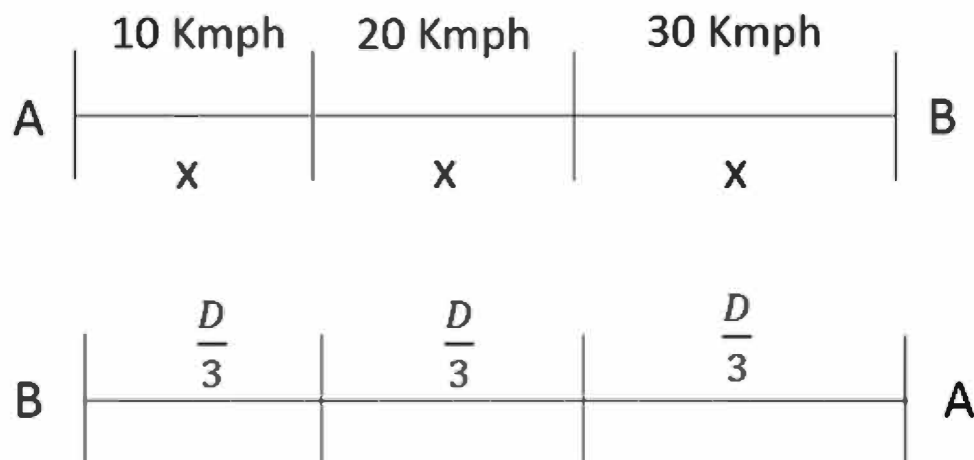
Previous year Quant's for CAT

CAT 2019

The percentage by which Bimal's travel time exceeds Amal's travel time is nearest to

- A.22
- B.19
- C.21
- D.20

Correct Answer. 22



Let the distance be = 60 Kms.

So, Bimal travels a distance of 20Kms in each mode and Amal travels 10, 20 and 30 Kms respectively in 1 hour each

Time taken by Bimal = Time taken to travel 20 kms in 10 Kmph + Time taken to travel 20 kms in 20 Kmph + Time taken to travel 20 kms in 30 Kmph
Time taken by Bimal = $2 + 1 + \frac{2}{3}$ hours = $3 + \frac{2}{3}$ hours

Extra time taken by Bimal = $\frac{2}{3}$ hour

Percentage increase in time = $\frac{2/3}{3} \times 100 = \frac{2}{9} \times 100 = 22$

So, Percentage increase in time = 22

Previous year Quant's for CAT

CAT 2019

Question 18:

Amala, Bina, and Gouri invest money in the ratio 3 : 4 : 5 in fixed deposits having respective annual interest rates in the ratio 6 : 5 : 4. What is their total interest income (in Rs) after a year, if Bina's interest income exceeds Amala's by Rs 250?

- A. 7000
- B. 6000
- C. 6350
- D. 7250

Correct Answer. 7250

	Amala	Bimala	Gouri
Amount invested	3	4	5
Interest rate	6	5	4

We know, Interest income Amount invested

And Interest income Interest rate

Therefore, Interest income must be in the ratio of the product of their Amount invested and Interest rate.

So, Ratios of Interest incomes of Amala, Bimala and Gouri = 18: 20: 20 Bina's Interest income exceeds Amala by 250 Rs.

So, $20x - 18x = 250$

$2x = 250$

So, Total Interest income = $250 \times (9 + 10 + 10) = 7250$ Rs.

Total Interest Income = Rs.7250

Previous year Quant's for CAT

CAT 2019

Question 19:

A chemist mixes two liquids 1 and 2. One litre of liquid 1 weighs 1 kg and one litre of liquid 2 weighs 800 gm. If half litre of the mixture weighs 480 gm, then the percentage of liquid 1 in the mixture, in terms of volume, is

- A. 70
- B. 85
- C. 80
- D. 75

Correct Answer. 80

In Liquid L1 - 1L = 1000 grams

So, 1000 ml = 1000 grams

1 ml = 1 gram

x ml = x grams

In Liquid L2 - 1L = 800 grams

1000 ml = 800 grams

1ml = 0.8 grams

(500-x) ml = (500-x) x 0.8 grams

Total mass = x + 400 - 0.8x = 480 grams

0.2x = 80 grams

x = 400 grams

Therefore, Liquid 1 has 400 ml and Liquid 2 has 500 - 400 = 100 ml. Therefore,
Percentage of Liquid 1 = $400/500 \times 100 = 80\%$

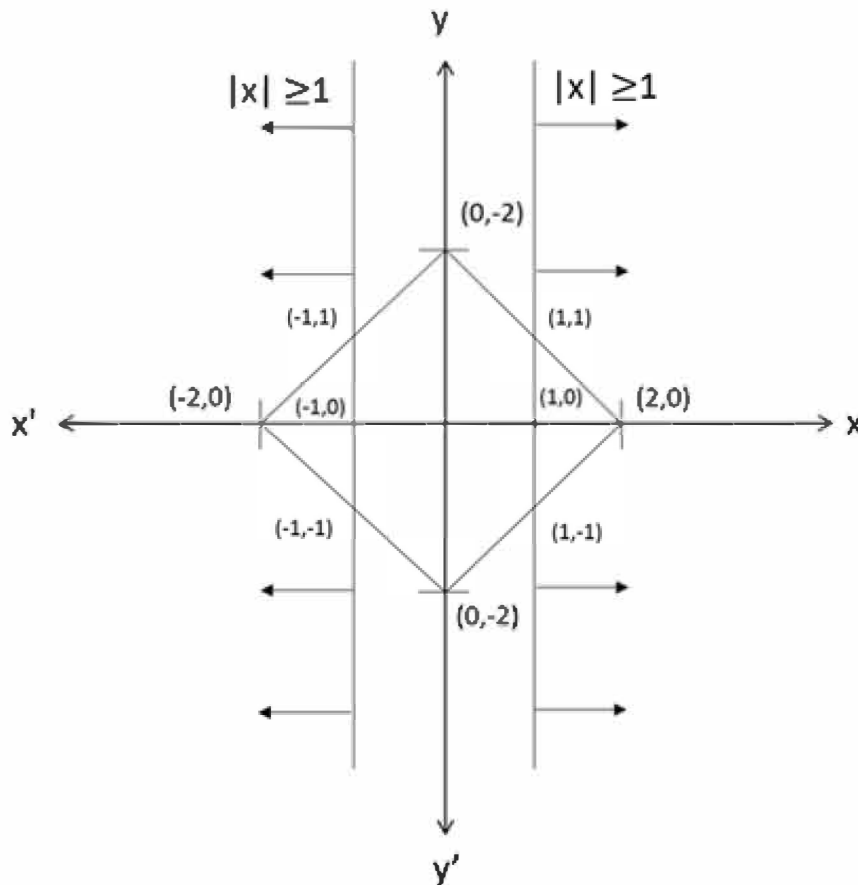
Question 20:

Let S be the set of all points (x,y) in the x-y plane such that $|x| + |y| \leq 2$ and $|x| \geq 1$. Then, the area, in square units, of the region represented by S equals [TITA]

Correct Answer. 2

Previous year Quant's for CAT

CAT 2019



Construct the given data on a rough graph.

Required area = Sum of area of two smaller triangles [(-2,0) (-1,1) (-1,-1) and (2,0) (1,-1) (1,1)]

Required area = $2 \times \frac{1}{2} \times \text{base} \times \text{height}$

Required area = $1 \times 2 = 2$ Sq units

Question 21 :

The number of solutions of the equation $|x|(6x^2 + 1) = 5x^2$ is [TITA]

Correct Answer. 5

Previous year Quant's for CAT

CAT 2019

$$|x| (6x^2 + 1) = 5x^2$$

We know that $|x^2| = x^2$

Let $y = |x|$

$$\text{So, } y^2 = x^2$$

$$\text{So, } y (6y^2 + 1) = 5y^2$$

$$6y^2 + 1 = 5y$$

$$6y^2 - 5y + 1 = 0$$

$$y = 1/3 \text{ or } 1/2$$

Since, $y = |x|$

$$y = 1/3 \text{ or } -1/3, 1/2 \text{ or } -1/2$$

Since we have cancelled y in our first step, $x = 0$ is also a solution

$$\text{So, Number of possible solutions} = 4 + 1 = 5$$

Question 22:

Three men and eight machines can finish a job in half the time taken by three machines and eight men to finish the same job. If two machines can finish the job in 13 days, then how many men can finish the job in 13 days? [TITA]

Correct Answer. 13

Let Machines be referred as R and men be referred as M

It is given that, three men and eight machines can finish a job in half the time taken by three machines and eight men to finish the same job. From the given data,

$$3M + 8R = 2 \times (3R + 8M)$$

$$2R = 13M$$

$$R = 13/2 M$$

Therefore, if two machines can finish a job in 13 days, 2 Robos can finish the job in 13 days.

